SOUND AND MUSIC.

SOUND AND MUSIC:

AN ELEMENTARY TREATISE ON THE
PHYSICAL CONSTITUTION OF

Musical Sounds and Harmony,

THE CHIEF ACOUSTICAL DISCOVERIES OF
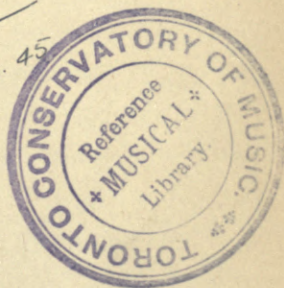## *PROFESSOR HELMHOLTZ,*

BY

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SECOND EDITION
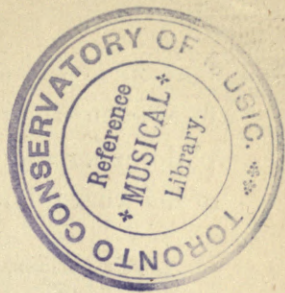
PREFACE TO THE FIRST EDITION.

THE following treatise, portions of which have been delivered in lectures at the South Kensington Museum, the Royal Academy of Music, and elsewhere, aims at placing before persons unacquainted with Mathematics an intelligible and succinct account of that part of the Theory of Sound which constitutes the physical basis of the Art of Music. No preliminary knowledge, save of Arithmetic and of the musical notation in common use, is assumed to be possessed by the reader. The importance of combining theoretical and experimental modes of treatment has been kept steadily in view throughout.

The author has incorporated the chief Acoustical discoveries of Professor Helmholtz, but adopted his own course in explaining them and developing their connection with the previously established

T.   *b*

portion of the subject. The present volume, therefore, even where its obligations to the great German philosopher are the deepest, is not a mere epitome of his work[1], but the result of independent study.

TRINITY COLLEGE, CAMBRIDGE,
    June, 1873.

[1] *Die Lehre von den Tonempfindungen. Dritte Ausgabe. Braunschweig.* 1870. Of this profound and exhaustive treatise it is not too much to say that it does for Acoustics what the *Principia* of Newton did for Astronomy.

PREFACE TO THE SECOND EDITION.

THE present edition is the result of a careful revision applied to its predecessor.

An unsatisfactory artificial hypothesis as to the motion of particles in the surface of water-waves, which was avowedly introduced into the first edition for the sake of simplicity, is replaced by a mode of treatment based upon observed facts.

Points on which special difficulty was experienced by readers of the first edition are here somewhat more fully elucidated.

The author has, of course, consulted the revised edition[1] of Helmholtz's *Tonempfindungen*.

TRINITY COLLEGE, CAMBRIDGE,
June, 1883.

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[1] *Vierte umgearbeitete Ausgabe. Braunschweig,* 1877.

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CHAPTER I.

ERRATUM.

Page 14, line 21, *for* movement *read* moment.

sation is greatly weakened, or, if originally but feeble, altogether extinguished. Persons whose auditory apparatus is malformed, or has been destroyed by disease, may be totally unconscious of any sound, even during a thunder-storm or the discharge of artillery. It would be entirely in accordance with the mode of action of our other senses if what we feel as Sound is represented, externally to ourselves, by a state of things very different to the sensation we experience. Analogy, then, indicates

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CHAPTER I.

ON SOUND IN GENERAL AND THE MODE OF ITS TRANS-
MISSION.

1. In listening to a sound, all that we are im-
mediately conscious of is a peculiar *sensation*. This
sensation obviously depends on the action of our
organs of hearing; for if we close our ears the sen-
sation is greatly weakened, or, if originally but feeble,
altogether extinguished. Persons whose auditory
apparatus is malformed, or has been destroyed by
disease, may be totally unconscious of any sound,
even during a thunder-storm or the discharge of
artillery. It would be entirely in accordance with
the mode of action of our other senses if what
we feel as Sound is represented, externally to our-
selves, by a state of things very different to the
sensation we experience. Analogy, then, indicates

T.   1

that some purely mechanical phenomena external to the ear will prove to be *turned into the sensation we call Sound* by a process carried on within that organ and the brain with which it is in direct communication. This mechanical agency, whatever may be its nature, is usually set going at a distance from the ear, and, to reach it, must traverse the intervening space. In doing so it can pass through solid and liquid as well as gaseous bodies. If one end of a felled tree is gently scratched with the point of a penknife, the sound is distinctly audible to a listener whose ear is pressed against the other end of the tree. When a couple of pebbles are knocked together under water, the sound of the blow reaches the ear after first passing through the intervening liquid. That Sound travels through the air is a matter of universal experience, and needs no illustration. In every case accessible to common observation where Sound passes from one point of space to another, it necessarily traverses *matter*, either in a solid, liquid or gaseous form. We may hence conjecture that the presence of a material medium of some kind is indispensable to the transmission of Sound. This important point can be readily brought to the test of experiment, as follows. Let a bell kept ringing by clockwork be placed under the receiver of an air-pump, and the air gradually exhausted. Provided

that suitable precautions are taken to prevent communication of Sound to the external air through the body of the receiver, the bell will appear to ring more and more feebly as the exhaustion proceeds, until at last it altogether ceases to be heard. While the air is being readmitted, the sound of the bell will gradually recover its original loudness. This experiment shows that Sound cannot travel *in vacuo*, but requires for its transmission a material medium of some kind. The air of the atmosphere is, in the vast majority of cases, the medium which conveys to the ear the mechanical impulse which that wonderful organ *translates*, as it were, into the language of Sound.

2. Having ascertained that a material medium acts in every case as the carrier of Sound, we have next to examine in what manner it performs this function. The roughest observations suffice to put us on the right track in this enquiry by pointing to a connexion between Sound and Motion. The passage through the air of sounds of very great intensity is accompanied by effects which prove the atmosphere to be in a state of violent commotion. The explosion of a powder-magazine is capable of shattering the windows of houses at several miles' distance. In the case of sounds of only ordinary loudness the accompanying air-motion manifests itself in no such

unmistakable way : a little attention will, however,
usually detect a certain amount of movement on the
part of the sound-producing apparatus, which is pro-
bably capable of being communicated to the sur-
rounding air.   Thus, a sounding pianoforte-string
can be both seen and felt to be in motion : the
movements of a finger-glass stroked on the rim by a
wet finger can be recognised by observing the thrills
which play on the surface of the water it contains :
sand strewed on a horizontal drum head is thrown off
when the drum is beaten.   These considerations raise
a presumption that Sound is *invariably* associated
with agitation of the conveying medium—that it is
impossible to produce a sound without at the same
time setting the medium in motion.   If this should
prove to be the case, there would be ground for the
further conjecture that motion of a material medium
constitutes the mechanical impulse which, falling on
the ear, excites within it the sensation we call Sound.
Let us try to form an idea of the *kind* of motion
which the conditions of the case require.

3.   It will be convenient to begin by determining
the *rate* at which Sound travels.   This varies, indeed,
with the nature of the conveying medium.   It will
suffice, however, for our present purpose to ascertain
its velocity in *air*, the medium through which the
vast majority of sounds reach our ears.   As long as

we confine our attention to sounds originating at
but small distances from us, their passage through
the intervening space appears instantaneous. If,
however, a gun is fired at a considerable distance,
the flash is seen before the report is heard—a proof
that an appreciable interval of time is occupied by
the transmission of the sound. The occurrence of
an echo, in a position where we can measure the dis-
tance passed over by the sound in travelling from
the position where it is produced to that where it
rebounds, gives us the means of measuring the velo-
city of Sound ; since we can, by direct observation,
ascertain how long a time is spent on the out-and-
home journey. The following easy experiment gives
a near approximation to the actual velocity of Sound
—in fact a much closer one than the rough nature of
the observation would have led one to expect. In
the North cloister of Trinity College, Cambridge,
there is an unusually distinct echo from the wall at
its eastern extremity. Standing near the opposite
end of the cloister, I clapped my hands rhythmically
at a rate such that the strokes and echoes were heard
alternately *at equal intervals of time.* A friend at
my side, watch in hand, counted the number of strokes
and echoes. The result was that there were 76 in half
a minute, *i.e.* 38 strokes and 38 echoes. A little con-
sideration will show that the sound traversed the

cloister and returned to the point of its origination
regularly once in each interval between a stroke and
its echo. Since each such interval was exactly equal
to that between an echo and the following stroke,
the whole movement of Sound took place in alternate
equal intervals, *i.e.* in half the observed time, or
fifteen seconds. Accordingly the sound travelled *to
and fro* in the cloister 38 times in 15 seconds. The
length thus traversed, I found to be 419 feet. The
velocity of Sound per second thus comes out equal to
$\dfrac{38 \times 419}{15}$, or 1061[1] feet and a fraction. Sound, then,
travels through the air at the rate of upwards of
1,000 feet in a second, which is more than 600 miles
an hour, or about 15 times the speed of an express
train. In solid and liquid bodies its velocity is still
greater, attaining in the case of steel-wire a speed of
from 15,000 to 17,000 feet in a second[2], or, roughly
speaking, about 200 times that of an express train.

4. Though the Sound-impulse advances with
a steady and high velocity, the medium by which
it is transmitted clearly does not share such a
motion. Solid conductors of Sound remain, on the

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[1] This is about 50 feet below its real value under the
circumstances of my observation. See Tyndall's *Sound*, Third
Edition, p. 23.

[2] Tyndall's *Sound*, p. 38.

whole, at rest during its passage, and a slight yielding of their separate parts is all that their constitution generally admits of. In fluids, or in the air, a continuous forward motion is equally out of the question. The movement of the particles composing the Sound-conveying medium will be found to be of a kind examples of which are constantly presenting themselves, but without attracting an amount of attention at all commensurate with their interest and importance.

5. An observer who looks down upon the sea from a moderate elevation, on a day when the wind, after blowing strongly, has suddenly dropped, sees long lines of waves advancing towards the shore at a uniform pace and at equal distances from each other. The effect to the eye is that of a vast army marching up in column, or of a ploughed field moving along horizontally in a direction perpendicular to the lines of its ridges and hollows. The *actual* motion of the water is, however, very different from its *apparent* motion, as may be ascertained by noticing the behaviour of a cork or other body floating on the surface of the sea, and therefore sharing its movement. The floating body does not advance with the waves, but rides over their crests and sinks into their troughs as though it were a buoy at anchor. Hence, while the waves travel

steadily forward horizontally, each of the fluid particles which compose them describes over and over again a fixed orbit of its own.

Thus, when we say that the waves advance horizontally, we mean, not that the masses of water of which they at any given instant consist advance, but that these masses, by virtue of the separate motions of their individual particles, *successively arrange themselves in the same relative positions,* so that the curved shapes of the surface, which we call waves, are horizontally transmitted *without their materials sharing in the progress.* The accompanying figure will show how this happens.

Fig. 1.

Let the full curved line *AB* represent a section of a part of the sea-surface at any given instant made by a vertical plane through the direction of wave-motion, and suppose that during, say, the next ensuing second of time, the separate fluid particles rearrange themselves by virtue of their respective orbital motions in such a manner that, at the end of that second, they constitute a curved line identical in shape and size with *AB*, and only

differing from it in horizontal position. Let the dotted line $A'B'$ represent the curve thus formed. As the two outlines AB and $A'B'$ are exactly alike, the joint effect produced by the separate particle-movements on the eye of a spectator is just what it would have been had we pushed the curve AB along horizontally until it came to occupy the position $A'B'$.

In order further to illustrate this point, let us suppose that a hundred men are standing in a line and that the first ten are ordered to kneel down : a spectator who is too far off to distinguish individuals will merely see a broken line like that in the figure below.

Now, suppose that after one second the eleventh man is ordered to kneel and the first to stand ; after two seconds the twelfth man to kneel and the second to stand ; and so on. There will then continue to be a row of ten kneeling men, but during each second it will be shifted one place along the line. The distant observer will therefore see a depression steadily advancing along the line. The state of things presented to his eye after twenty, sixty and ninety seconds, respectively, is shown in Fig. 2.

Fig. 2
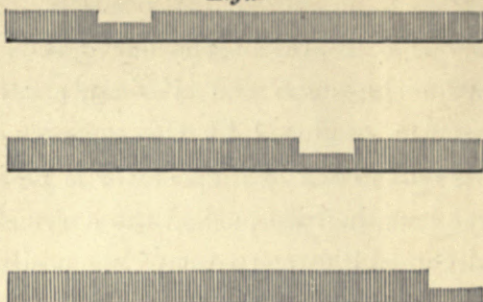
There is here no horizontal motion on the part
of the men composing the line, but their vertical
motions give rise, in the way explained, to the hori-
zontal transference of the depression along the line.
The reader should observe that for no two con-
secutive seconds does the kneeling row consist of
exactly the same men, while in such positions as
those shown in the figures, which are separated by
more than ten seconds of time, the men who form it
are totally different.

6.   Let us now return to the sea-waves and
examine more closely the elements of which they
consist.

Fig. 3 represents a vertical section of one com-
plete wave.

Fig. 3
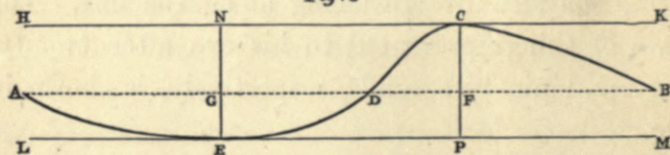
The dotted line is that in which the horizontal plane forming the surface of the sea when at rest cuts the plane of the figure; it is called the *level-line*. The distance between the two extreme points of the wave, measured along this line, is called the *length of the wave*. C is the highest point of the crest DCB; E the lowest point of the trough AED. CF and EG are vertical straight lines through C and E; HCK and LEM are horizontal straight lines through the same two points. The vertical distance between the lines HK and LM is called the *amplitude* of the wave. Thus AB is the length of the wave, and, if we produce EG and CF to cut the lines HK and LM in N and P respectively, we have, for its amplitude, either of the equal lines EN, PC. Each of these is clearly equal to FC and GE together, that is to say the amplitude of the wave is equal to the height of the crest above the level-line together with the depth of the trough below it. In addition to the length and amplitude of the wave, we have one more element, its *form*. The wave in the figure has its crest shorter than its trough and higher than its trough is deep. Moreover the part DC of the crest is steeper than the part CB, while in the trough the parts AE and ED are nearly equally steep. Sea-waves have infinitely various shapes dependent on the character of the

wind which originated them and on the depths and configuration of the sea bottom over which they pass. Hence, before we can lay down a wave in a figure, we must know the nature of the wave's curve, *i.e.* its form.

Since the crests of the waves are raised above the ordinary level of the sea, the troughs must necessarily be depressed below it, just as, in a ploughed field, the earth heaped up to form the ridges must leave empty the furrows from which it was taken. Each crest being thus associated with a trough, it is convenient to regard one crest and one trough as forming together one complete wave. Thus each wave consists of a part raised above, and a part depressed below, the horizontal plane which would be the surface of the sea were it not being traversed by waves.

7. The *length, amplitude* and *form* of a wave completely determine the wave, just as the length, breadth and height of an oblong block of wood, *i.e.* its three dimensions, fix the size of the block. These three elements of a wave are mutually independent, that is to say we may alter any one of them without altering the other two. This will be seen by a glance at the accompanying figures.

(1) shows variation in *length* alone; (2) in *amplitude* alone; (3) in *form* alone.

Fig. 4 (1)

(2)   (3)
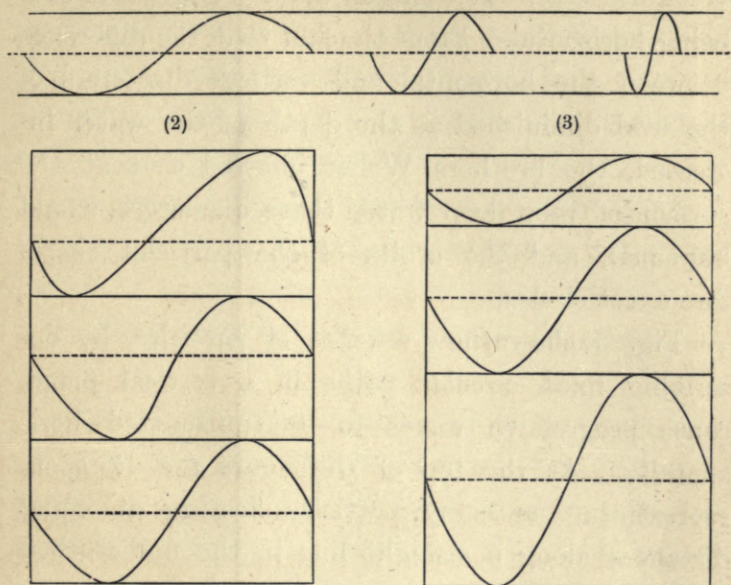
8. We will next examine more closely the separate movements of particles at the surface of water, and their joint result the transmission of waves. The brothers Heinrich and Wilhelm Weber published, in 1825, a series of very careful observations on water-waves passing along troughs with glass sides not unlike very long and very narrow aquariums. By ingeniously planned arrangements, which it would take too long to describe here, they ascertained that each particle on the surface continued, during the passage of a succession of equal waves, to describe a fixed oval path in a

vertical plane, the longer diameter of the oval
being horizontal. From the fact that the difference
between the horizontal and vertical diameters of
the oval diminished as the depth of the water in-
creased, the brothers Weber inferred that at the
surface of very deep water these diameters would
be equal, and the orbits of the particles there-
fore exact circles.

Fig. 5 shows how a series of particles, by de-
scribing fixed circular paths in a vertical plane,
cause progressive waves to be transmitted hori-
zontally. At the top of the figure are 17 spots
representing as many particles floating at equal
distances along a straight line in the undisturbed
surface of very deep water indicated by dots in the
figure. The equal vertical circles are the paths
which the particles are respectively about to de-
scribe in the direction in which the hands of a
watch move, as indicated by a curved arrow in the
figure, and with equal uniform velocities.

(0) represents the state of things at a moment
when the group of particles plot out two complete
equal waves included by the brackets *A* and *B*, each
consisting of a crest to the right and trough to the
left as marked by the sub-brackets *a*, *a'* and *b*, *b'*
respectively. The horizontal dotted line represents,
as before, the section of the undisturbed surface or

level-line. The reader will at once observe, on looking along the line of circles from left to right, that each particle occupies in its own orbit a position one-eighth of a revolution behind that of the particle to the left of it.

(1) shows the arrangement of the particles when an interval of time equal to one-eighth of the period of a complete circle-revolution has elapsed from the moment at which they were grouped as in (0).

Similarly (2) represents the state of things after two-eighths of that period, (3) after three-eighths of it, and so on, until in (8), one complete period of revolution having in all elapsed, the particles are necessarily again in the same positions as those shown in group (0).

If now the reader allows his eye to be guided by the oblique dotted lines of the figure, he will readily perceive that the wave A has moved horizontally to the right at each of the above stages and, after the lapse of one complete period of particle-revolution occupies, in (8), the position which the wave B held in (1), *i.e.* has traversed its own wave-length from left to right. We may express this result generally by saying that *while an individual particle performs one complete orbital revolution, the wave advances by one wave-length.* This is the fundamental pro-

position of wave-motion, and should be carefully mastered and remembered. In the figure employed to demonstrate it, a wave has been only roughly plotted out by a small number of spots, and its movements estimated at but a few arbitrarily chosen instants. By increasing the numbers of these two elements, however, we might make an indefinitely near approach to continuity both of form and of motion.

9. The characteristic phenomenon of wave-motion, viz. *an apparent forward movement unshared by the materials which give rise to it,* though most frequently seen on the surface of water, is by no means confined to fluid bodies. When a carpet is being shaken, bulging forms exactly like water-waves are seen running along it. A flexible string, one end of which is tied to a fixed point and the other held in the hand, exhibits the same phenomenon when the loose end is sharply jerked aside. It has accordingly been found convenient to extend the term wave in order to designate as 'wave-motion' any movement coming under the definition just given. We proceed to an instance in which the individual particles, instead of describing circular orbits move to-and-fro, or *vibrate,* in straight lines perpendicular to the direction of wave-propagation.

Before applying to this case the method followed in Fig. 5 it will be desirable to define what is meant

Fig. 5.
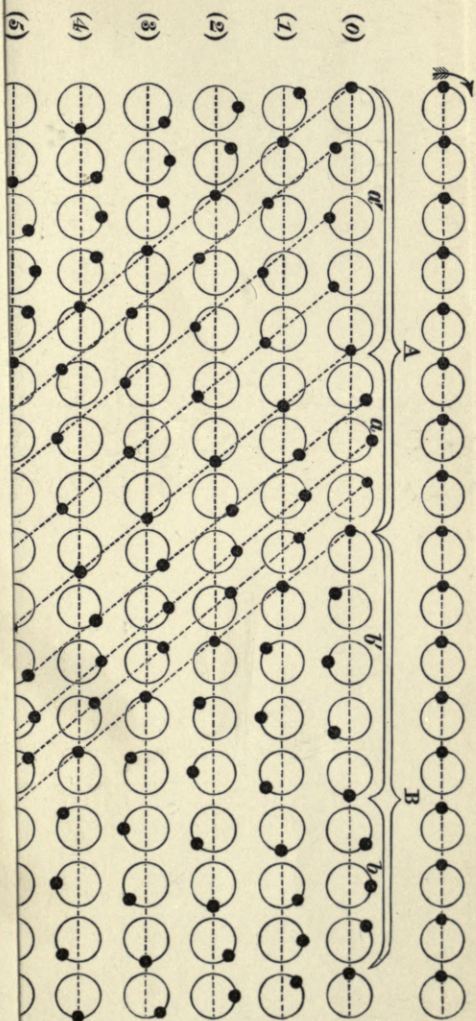
by a 'complete vibration' as that term is used by English writers.

Let a point move from A' to A along the straight

*Fig.*6

A

O

A'
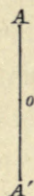
line $A'A$ and then back again to A'. A French writer would describe this movement as made up of *two* complete vibrations executed in opposite directions. An English writer would designate it as *one* complete vibration consisting of two half-vibrations performed in opposite directions. This latter usage will be adhered to in the sequel. The completion of a vibration is thus to be determined by the moving point having traversed its entire path, *in both directions*, and this is equally true if instead of starting from either extremity it occupies initially an intermediate position such as O and moves from O to A, from A to A' and from A' to 0 ; or reversely from O to A', from A' to A and from A to O.

The 'period' of a vibration accordingly means the time which it takes the moving point to perform one such complete cycle of its motion.

Let the 17 spots lying equidistantly along the
dotted straight line at the head of Fig. 7 represent
as many particles of an elastic string stretched
between fixed points of attachment not shown in
the figure. These particles are about to vibrate in
straight lines perpendicular to the direction of the
string. (0) shows the state of things when the
particles plot out two complete waves *A* and *B*.
The distinction between 'crest' and 'trough' has
now disappeared. All that can be said is that each
wave is formed of two protuberances lying on oppo-
site sides of the undisturbed position of the string,
which replaces the 'level-line' of the water-waves
and is, like it, represented by a dotted line in the
figure.

By looking along 0 from left to right the reader
will notice that each particle occupies in its own path
a position one-eighth of a period of vibration behind
that of the particle next to the left of it. (Compare
p. 15). (1), (2), (3)...show the positions of the par-
ticles after one-eighth, two-eighths, three-eighths...
of a complete period of particle-vibration. By follow-
ing any one of the vertical lines of spots it will be
seen that in the instance selected for this figure,
each particle moves more rapidly in the neighbour-
hood of its undisturbed position than it does near the
extremities of its swing. In (8) the particles have

Fig. 7
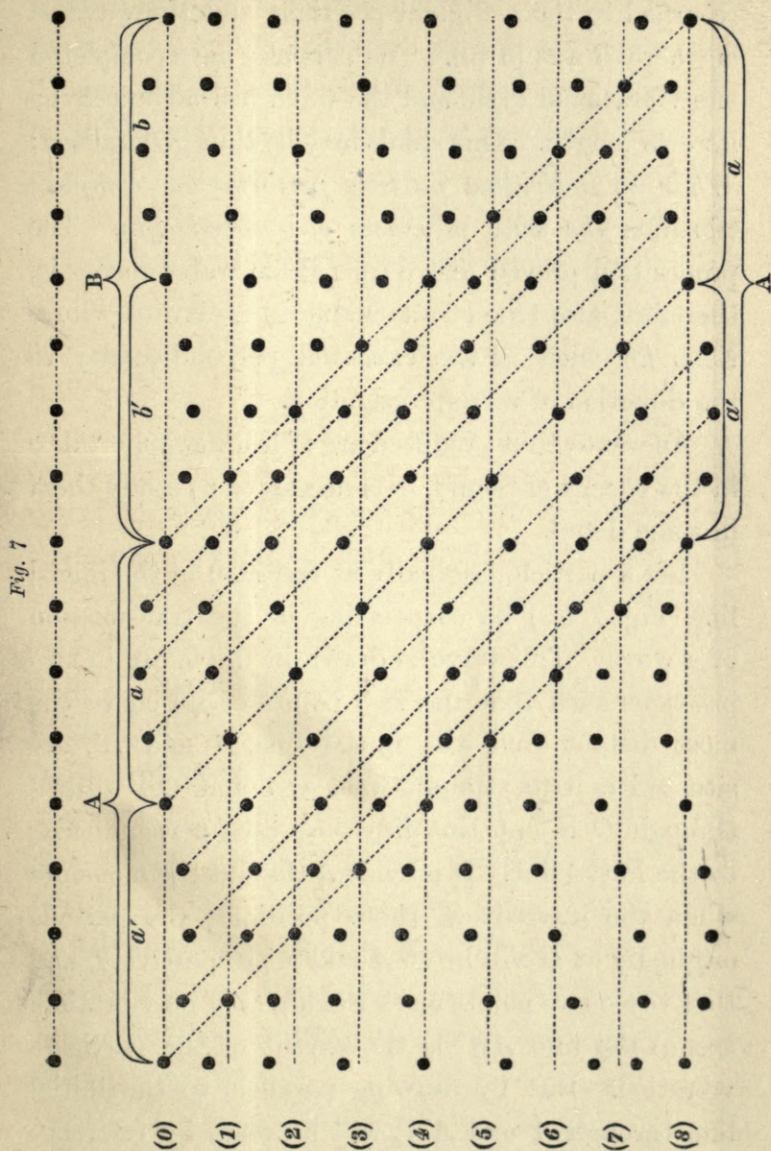
returned to their original positions and the wave A is where B was in (0). The particles have completed one vibrational cycle and the wave has advanced by its own length. This result may be thus generalised: *While an individual particle performs one complete vibration the wave advances one wave-length.* The proposition proved above (p. 15) for water-waves is, therefore, also true of waves due to transverse vibrations *i.e.* such as are executed perpendicularly to the direction of wave-propagation.

As waves thus produced are of leading importance in the theory of Sound, it is necessary to study them in some detail.

Let a particle originally at rest at 0 in the initial line (Fig. 7 *bis*) be cooperating in the transmission of a wave This wave is drawn in the figure in two positions such that the two points of its curve the most distant from the initial line, A and A', are situated in two straight lines OA and OA' drawn through O in opposite directions each perpendicular to the initial line. It is evident that, at the moments when the wave is in these positions, the particle originally at O will be at A and A' respectively, and that these two points mark the limits of its vibration. Hence the line AA' is the extent of the particle's vibration. But by drawing parallels to the initial line through A and A' it will be seen, by reference

to the definition in § 6, that AA' is also the amplitude of the wave. 'Extent of particle-vibration' and 'amplitude of corresponding wave' are, therefore, only different ways of expressing the same thing.

Fig. 7 (bis)
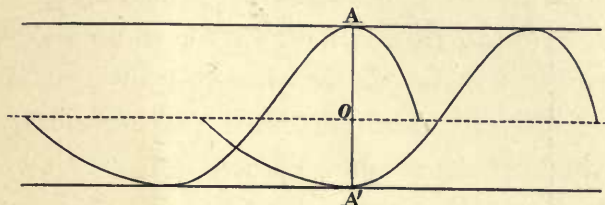
10.   When a series of continuous equal waves are being transmitted, each particle, after completing one vibration, will instantly commence another precisely equal vibration, and go on doing so as long as the transmission of waves is maintained. This kind of motion, in which the same movement is continuously repeated in successive equal intervals of time, is called 'periodic,' and the time which any one of the movements occupies is called its 'period.' Thus, to continuous equal waves correspond continuous periodic particle vibrations.

We will next compare the periods of the vibrations which produce waves of *different lengths* advancing at the same speed.

In Fig. 8, waves of three different lengths are

represented.   One wave of (1) is as long as two of
(2), and as three of (3).   Therefore, in virtue of the
fact proved in § 9, a particle makes *one* complete
vibration in (1) while the long wave passes from A
to B, *two* in (2) while the shorter waves there pre-

Fig. 8
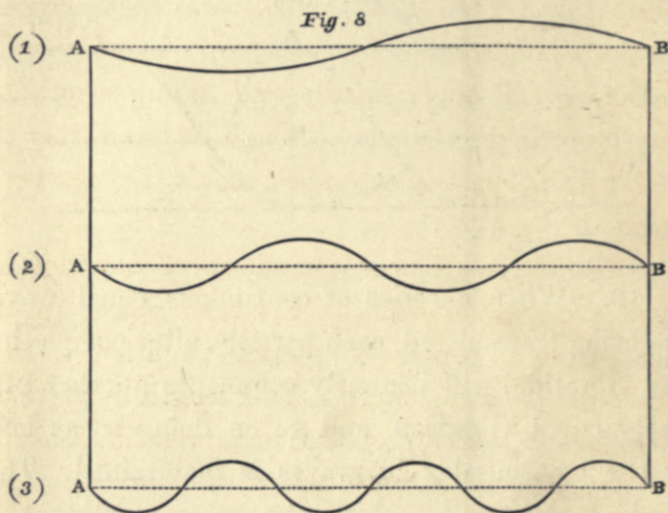
sented pass over the same distance, and *three* in the
case of the shortest waves of (3).   But the velocities
of these waves being by our supposition equal, the
times of describing the distance AB will be the same
in (1), (2) and (3).   Hence a particle in (2) vibrates
twice as rapidly, and in (3) three times as rapidly,
as in (1); or conversely, vibration in (1) is half as
rapid as in (2), and one-third as rapid as in (3).

The *rates* of vibration in (1), (2) and (3), (by

which we mean the numbers of vibrations performed in any given interval of time) are, therefore, proportional to the numbers 1, 2 and 3, which are themselves inversely proportional to the wave-lengths in the three cases respectively. We may express our result thus ; *The rate of particle-vibration is inversely proportional to the corresponding wave-length.* Similar reasoning will apply equally well to any other case ; the proposition, therefore, though deduced from the relations of particular waves, holds for waves in general.

The converse proposition admits of easy independent proof as follows. It has been shown (p. 20) that in one period of particle-vibration a wave traverses its own length. This length must therefore, if the velocity of the wave remain constant, be proportional to the period, *i.e. inversely proportional to the rate of vibration.*

11. We have now connected the extent of the particle-vibration with the amplitude, and its rate with the length, of the corresponding wave. It remains to examine what feature of the vibratory movement corresponds to the third element, the form of the wave.

Fig. 9.

B   O   A
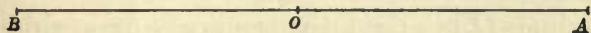
Suppose that two boys start together to run a

race from O to A, from A to B, and from B back to O, and that they reach the goal at the same moment. They may obviously do this in many different ways. For instance, they may keep abreast all through, or one may fall behind over the first half of the course and recover the lost ground in the second. Again, one may be in front over OAO, and the other over OBO, or each boy may pass, and be passed by, his competitor repeatedly during the race. We may regard the movement of each boy as constituting one complete vibration, and thus convince ourselves that a vibratory motion *of given extent and period* may be performed in an indefinitely numerous variety of *modes.* Let us now compare the positions of a particle at the expiration of successive equal intervals of time, when cooperating in the transmission of waves of different forms.

In each of the three cases in Fig. 10 the front of a wave is shown in the positions it respectively occupies at the end of ten equal intervals of time during which its intersection with the level-line moves from O through the equidistant points 1, 2, 3, 4, &c. of the initial line.

A particle whose place of rest is O will necessarily assume corresponding positions in the vertical line OA : thus the points where this line cuts the successive wave-fronts show the positions

of the vibrating particle at equal intervals of time.

Fig. 10
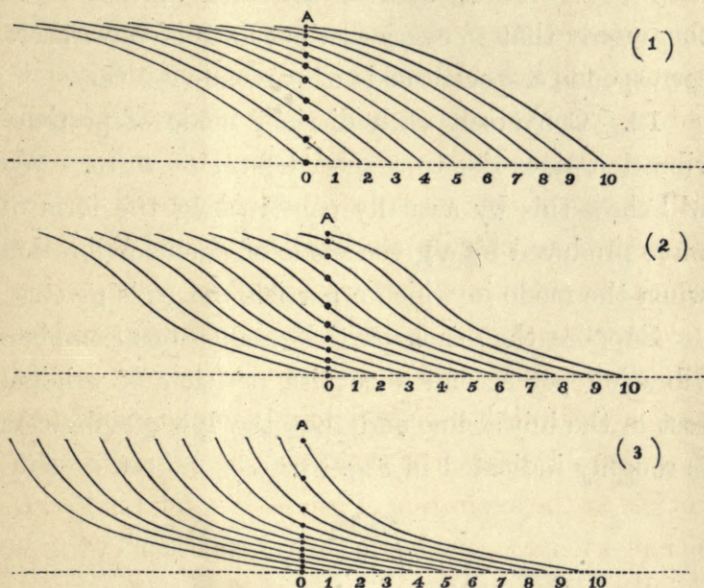
On comparing the three cases it will be seen that the mode of the particle's vibration is distinct in each. In (1), it moves fastest at O, and then slackens its pace up to A. In (2), it starts more slowly than in (1), attains its greatest speed near the middle of OA, and again slackens on approaching A. In (3), the pace steadily increases from O to A. The different wave-fronts shown in the figure have been purposely constructed with the same amplitude and length, in order that only such variations as were due to differ-

ences of form might come into consideration. The reader should construct for himself similar figures with other forms, and so convince himself more thoroughly that to every distinct form of wave there corresponds a special mode of particle-vibration.

12. Conversely each distinct mode of particle-vibration gives rise to a special form of wave. We will show this by actually constructing the form of wave produced by a given mode of particle-vibration when the mode in which a particle moves is given.

Suppose that each particle makes one complete vibration per second about its position of original rest in the initial line and that the law of vibration is roughly indicated in Fig. 11.

Fig.11
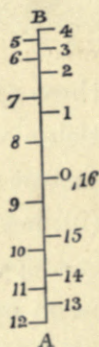
AB is the path described by a particle; *O* its position when in the initial line; 1, 2, 3, 4, 5, 6...12, 13...16 its positions after 16 equal intervals of time

each one-sixteenth of a second : 16 coincides with O, as the particle has returned to its starting-point.

Next, select a series of particles originally at rest in equidistant positions along the initial line, and so situated that each commences a vibration identical with that above laid down in Fig. 11 one-sixteenth of a second after the particle to the left of it has started on an equal vibration. Fig. 12 shows the rest-positions of the series of particles

$$a_0, \; a_1, \; a_2, \; a_3 \ldots a_{16},$$

in the initial line, and their contemporaneous positions

$$a_0, \; a_1', \; a_2', \; a_3' \ldots a_{16},$$

during a subsequent vibration.

Fig. 12.
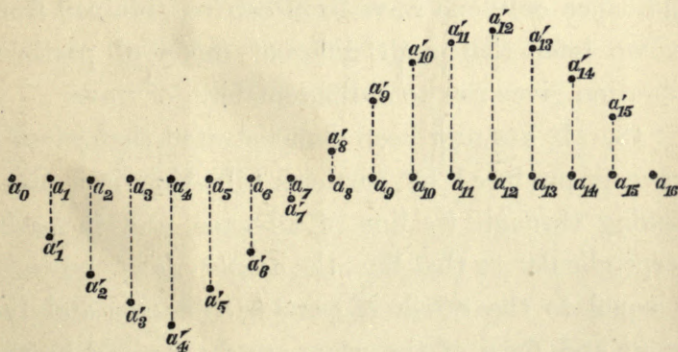
The moment selected for the figure is that in which the first of the series, a_0, is on the point of moving vertically upwards out of the initial line.

Since the second particle started one-sixteenth of a second after the first, its position in the figure will be below the initial line at a_1' making the line $a_1 \, a_1'$ equal to the line $O15$ in Fig. 11. The next particle, which is two-sixteenths of a second behind a_0 in its path, will be at a_2' making $a_2 \, a_2'$ equal to $O14$ in the same figure. In this way the positions of all the points $a_1' \, a_2' \, a_3'$, &c., in Fig. 12 are determined from Fig. 11. They give us, at once, a general idea of the form of the resulting wave. By laying down more points along the line AB in Fig. 11, we can get as many more points on the wave as we please, and should thus ultimately arrive at a continuous curved line. This is the wave-form resulting from the given vibration-mode with which we started, and, since only one wave-form can be obtained from it, we infer that each different mode of particle-vibration gives rise to a different form of wave.

13. It has now been demonstrated that when a wave is produced by particles vibrating in a plane passing through its line of advance, and in paths perpendicular to that line, the *amplitude* of the wave is equal to the *extent* of particle-vibration and the *length* and *form* of the wave are determined by the *rate* and *mode* of vibration respectively. These relations were also shown to hold conversely.

We will next consider a type of oscillatory

movement which is important from its similarity to that to which the transmission of Sound is due.

14. Any one who has looked down from a slight elevation upon a field of standing corn on a gusty day, must have frequently observed a kind of thrill running along its surface. As each ear of corn is capable of only a slight swaying movement, we have here necessarily an instance of *wave-motion*, the ear-vibrations corresponding to the particle-vibrations in the cases already examined. There is, however, this important peculiarity in the instance now before us, that the ears' movements are mainly horizontal, *i.e. executed in the line of the wave's advance.* The wave, therefore, no longer presents itself to the eye as exclusively a condition of alternate protuberance of outline in opposite directions, but mainly as a state of more tightly, or less tightly, packed ears. The

Fig 13.
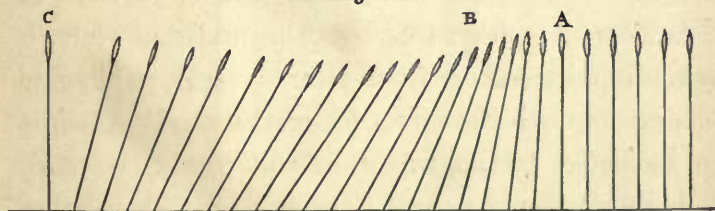
annexed figure gives a rough idea how this takes place. The wind is supposed to be moving from left to right and to have just reached the ear *A*. Its neighbours to the right are still undisturbed. The

stalk of *C* has just swung back into its erect position. The ears about *B* are closer to, and those about *C* further apart from, each other than is the case with those on which the wind has not yet acted.

After this illustration, it will be easy to conceive a kind of wave-motion in which there is no longer *any* movement transverse to the direction in which the wave is advancing.

15.   Let a series of particles, originally at rest in equidistant positions along a straight line, as in that at the head of Fig. 14, be executing equal periodic vibrations in that line, in such a manner that each is the same fixed amount further back in its path than is its neighbour to the left, and therefore exactly as much more forward than is its neighbour to the right.

(0) shows the condition of the row of particles at the moment when that on the extreme left is beginning its swing from left to right, which, in accordance with the direction of the wave's advance as indicated by an arrow in the figure, we may call its forward swing. The equidistant vertical straight lines fix the extent of vibration for each separate particle. The constant amount of relative backwardness or 'retardation,' as it is technically called, between successive particles is, as in Figs. 5

and 7, one-eighth of the period of a complete vibration. Thus, proceeding from left to right along the line (0), we have the first particle beginning a forward swing, the second, third, fourth and fifth particles entering respectively on the fourth, third, second and first quarters of a backward swing, and the sixth, seventh, eighth and ninth particles on the fourth, third, second, and first quarters of a forward swing.

Since the ninth particle is just beginning a forward swing, its situation is exactly the same as that of the first. Beyond this point, therefore, we have only repetition of the state of things between the first and ninth particles. The row (0) is therefore made up of a series of groups, or cycles, of the same number of particles arranged in the same manner throughout. Two such cycles, included by the large brackets *A* and *B*, are shown in (0). Each cycle is divided by the sub-brackets *a*, *a′* and *b*, *b′* into two parts. In *a* and *b* the distances between successive particles are less than, and in *a′* and *b′* greater than, the corresponding distances when the particles occupied their undisturbed positions. The cycles correspond to complete waves on the surface of water, the shortened and elongated portions of each cycle answering to the crest and trough of which each water-wave consists.

(1) shows the state of the row of particles when an interval of time equal to one-eighth of the period of a complete particle-vibration has elapsed from the moment shown in (0). The wave *A* has meantime advanced by one-eighth of its own wave-length into the position pointed to by the dotted lines.

The following rows (2), (3), (4), &c., show the state of things when two-eighths, three-eighths, four-eighths, &c., of a vibration-period has elapsed since (0). In each successive row the wave *A* is further advanced by one-eighth of its wave-length.

In (8), a whole vibration-period has elapsed since (0). Accordingly each particle has performed one complete vibration, and returned to the position which it held in (0). The wave *A*, meanwhile, has travelled constantly forward so as to be, in (8), where *B* was in (0), *i.e.* to have advanced by one whole wave-length. Hence, while any particle performs a complete vibration the wave advances by its own wave-length. Accordingly, for waves due to longitudinal vibrations, *i.e.* such as are executed in the line of wave-advance, a corresponding proposition holds good to that established in § 9 p. 20, for waves due to vibrations performed perpendicularly to the direction of wave-motion.

16. In the waves shown in Fig. 14, the particles in the bracket *a* are mutually equidistant, as are

Fig. 14.
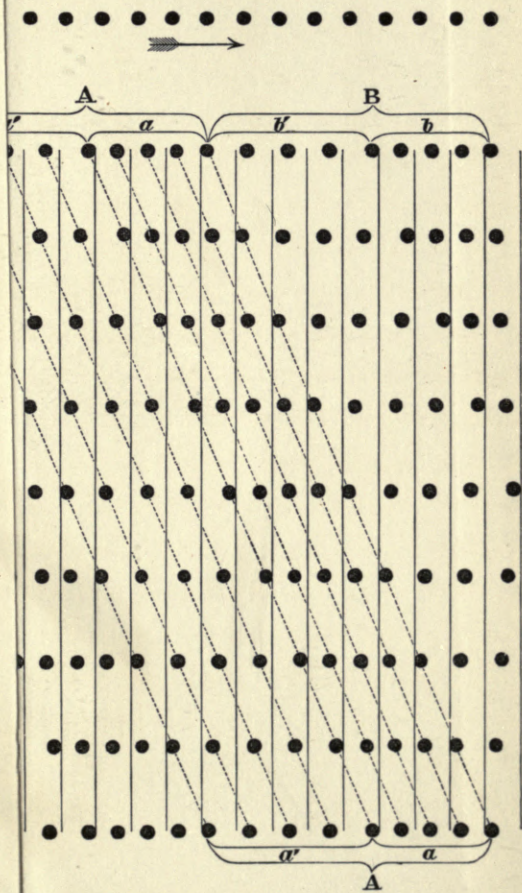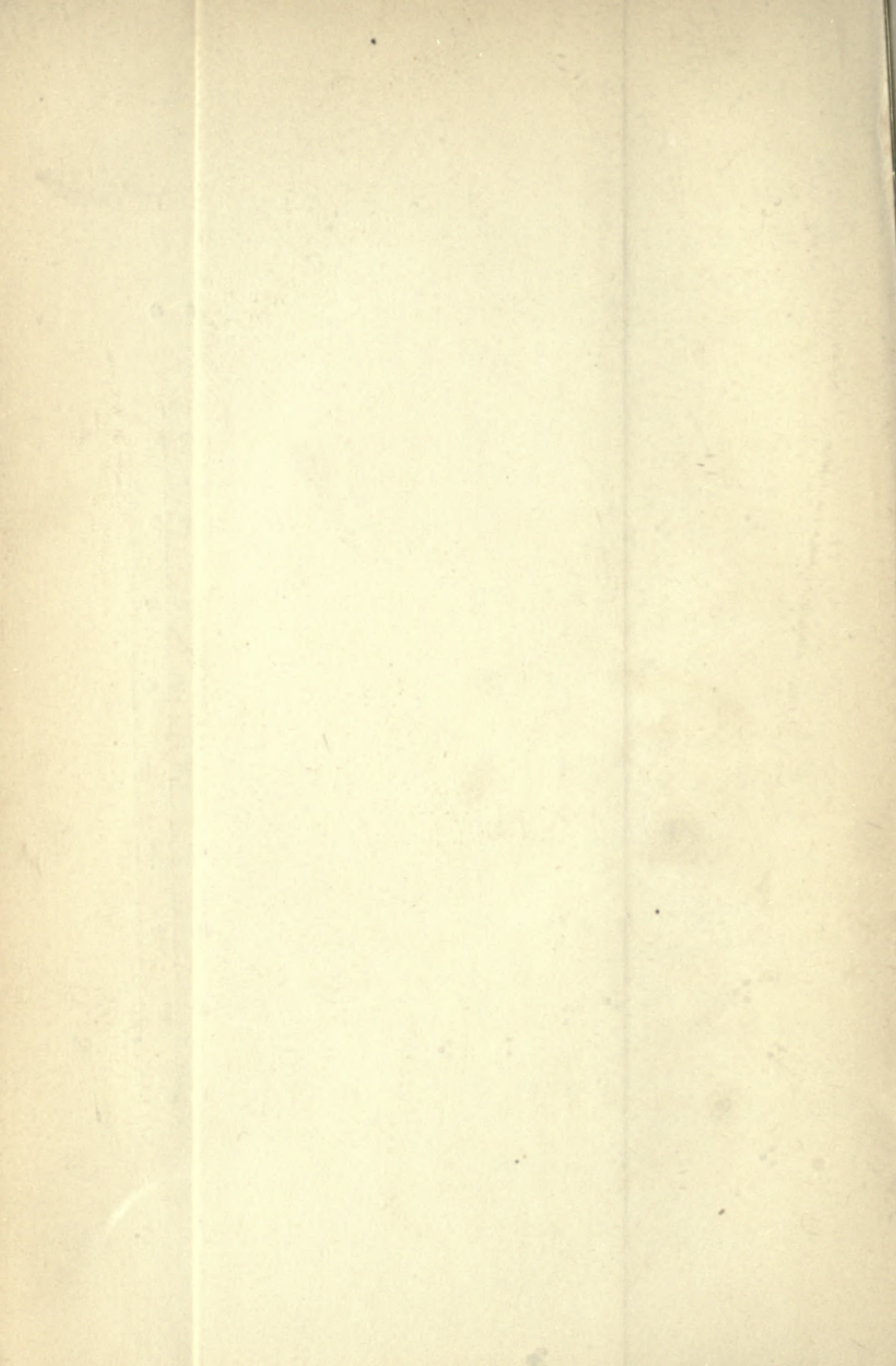
also those in the bracket *b*. This is due to the fact that, in the case there represented, the vibrating particles move uniformly, *i.e.* with equal velocity, throughout their paths. If we take other modes of vibration, we shall find that this equidistance no longer exists. Fig. 15 shows three distinct modes of vibration with the wave resulting from each, on the plan of Fig. 14. The extent of vibration and length of wave are the same in the three cases.

In (I) the particles move quickest at the middle, and slowest at the ends of their paths; in (II) fastest at the ends and slowest in the middle; in (III) slowest at the left end, and fastest at the right.

The shortest distance separating any two particles contained in *a* is, in (I), that between 7 and 8; in (II), that between 8 and 9; in (III), that between 5 and 6. The corresponding greatest distances are, in (I), between 2 and 3; in (II), between 1 and 2; in (III), between 4 and 5. The remaining particles likewise exhibit differences of relative distance in the three cases. Thus, the positions of greatest shortening and greatest lengthening occupy different situations in the wave, and the intermediate variations between them proceed according to different laws, when the modes of particle-vibration are different. The more particles we lay down in

T.   3

their proper positions in a and a', the less abrupt will be the changes of distance between neighbouring

Fig.15.
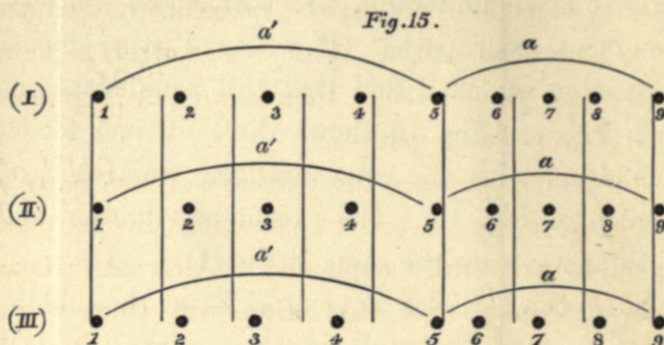
particles. By indefinitely increasing the number of vibrating particles, we should ultimately arrive at a state of things in which perfectly continuous changes of shortening and lengthening intervened between the positions of maximum shortening and maximum lengthening in the same wave.

17. Let us now replace our row of indefinitely numerous, but mutually unconnected, particles by the slenderest filament of some material whose parts (like those of an elastic string) admit of being compressed or dilated at pleasure. When any portion of the filament is shortened, a larger quantity of material is forced into the space which was before occupied by a smaller quantity. The matter within this space is therefore more tightly packed, more *dense*, than it was, *i.e.* a process of *condensa-*

tion has occurred. On the other hand, when a portion of the filament is lengthened, a smaller quantity is made to occupy the space before occupied by a larger quantity. Here the matter is more loosely packed, more *rare*, than it was, *i.e.* a process of *rarefaction* has taken place.

Let us now suppose the particles of the filament to be thrown into successive vibrations in the manner already so fully explained. Alternate states of condensation and rarefaction will then travel along the filament. It will be convenient to call these states 'pulses'—of condensation or rarefaction as the case may be. A pulse of condensation and a pulse of rarefaction together make up a complete wave.

18. The degree of condensation, or rarefaction, existing at any given point of a wave has been shown to depend on the mode in which the particles of the filament vibrate. It is therefore desirable to have some simple method, appealing directly to the eye, of exhibiting the law of any assigned mode of vibration. We may arrive at such a method by the following considerations.

When a line of particles vibrate longitudinally, they give rise to alternate pulses of condensation and rarefaction; when 'transversely,' they produce alternate protuberances on opposite sides of the line

of particles in their positions of rest.   Nevertheless,
if the vibrations in the two cases are identical in
all   respects   save   direction   alone,   the   distance
which, at any moment, separates an assigned par-
ticle from its position of rest will be the same
whether the vibrations are longitudinal or transverse.
It is therefore only necessary to construct the wave
corresponding to any system of *transverse* vibrations,
in the way shown in § 12, in order to obtain the
means of fixing the position of an assigned particle
at any given moment for the same system of vibra-
tions executed *longitudinally*.
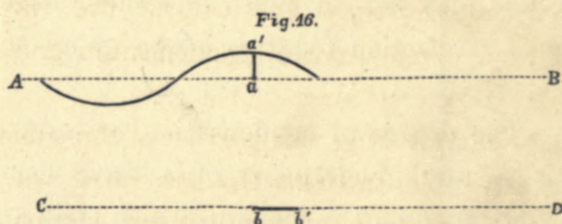
Fig. 16.

Let *AB* and *CD*, Fig. 16, be lines of particles
executing vibrations transverse to *AB* and along *CD*
respectively.   Let *a* and *b* be corresponding particles
in their positions of rest.   Draw the transverse wave
for any given instant of time : the particle originally
at *a* will now be at *a'*, and that originally at *b* will
be at *b'*, if *bb'* be made equal to *aa'*.

By performing the same process for different
instants, we can find as many corresponding posi-

tions of the longitudinally-vibrating particle as we
please. It is true that we learn *nothing new* by this,
since we cannot construct the wave-curve without
knowing beforehand the mode of the particle's vibra-
tion [§ 12]. Still, when we are dealing with
longitudinal particle-vibrations, and require to know
the law of the variation of condensation and rarefac-
tion at different points of a single wave, it is
convenient to have a picture of the mode of vibration
by which, as we know [§ 16], that law is determined.
Such a picture we have in the form of the wave
produced by the same mode of vibration when
executed transversely. Let us call the wave so
related to a given wave of condensation and rare-
faction its *associated wave.*

19. Before leaving this portion of the subject, it
will be advisable to draw the associated wave for
that particular mode of longitudinal vibration in
which each particle moves as if it were the extremity
of a pendulum traversing a path which is very short
compared to the pendulum's length. The meaning
of this limitation will be easily seen from Fig. 17.

Let O be the fixed point of suspension; OA the
pendulum in its vertical position; AB a portion of
a circle with centre O and radius OA; a, b, c, d,
points on this circle; AD a horizontal straight line
through A; aa', bb', cc', dd' verticals through a, b,

c, *d*, respectively. If the pendulum is placed in the
position *Oa*, and left to itself, it will reach an equally

Fig. 17.
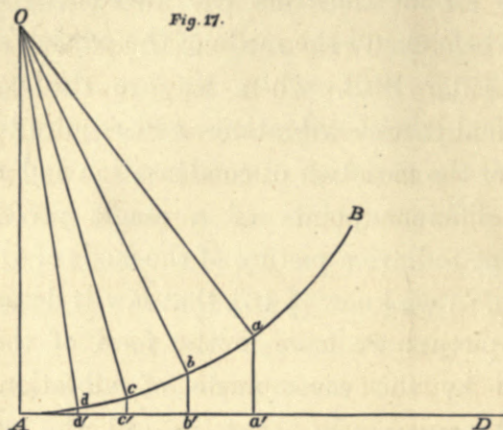
inclined position on the other side of *OA*, *i.e.* will
swing through twice the angle aOA before it turns
back again. Similarly, if started at *Ob* it will swing
through twice the angle *bOA* ; if at *Oc*, through
twice the angle *cOA* ; and so on. Now the extremity
of the pendulum, when at *a*, is further from the
horizontal line, *AD* than when it is at *b*, since *aa'*
is greater than *bb'* ; and when at *b* further than
when at *c*. If we make the pendulum vibrate through
only a small angle, by starting it, say, in the position
Od, its extremity will throughout its motion be very
near to the horizontal straight line *AD*. If we
make the angle small enough, or, which is the same
thing, take *Ad* sufficiently small compared with *OA*,

we may without any perceptible error suppose the
end of the pendulum to move in a horizontal straight
line instead of in a circular arc, *i.e.* along $d'A$
instead of dA.   To take an actual case, let us suppose
the pendulum to be 10 ft. long, and its extent of
swing 1 inch on either side of its vertical position.
An easy geometrical calculation shows that the
end of the pendulum will never be as much as
$\dfrac{1}{200}$ th of an inch out of the horizontal straight line
drawn through it in its lowest position.   This is a
vanishing quantity compared to the length of the
pendulum; we may, therefore, safely regard the
vibration as performed along $d'A$ instead of dA.
Such a vibration, though executed in a straight
line instead of in the arc of a circle, may be pro-
perly called a *pendulum-vibration*, as expressing the
law according to which it takes place.   This law
admits of simple geometrical illustration as follows.
Let a ball, or other small object, be attached to some
part of a wheel revolving uniformly about a fixed
horizontal axis, so that the ball goes round and round
in the same vertical circle with constant velocity.
If the sun is in the zenith, *i.e.* in such a position that
the shadows of all objects are thrown vertically, *the
shadow of the ball on any horizontal plane below it
will move exactly as does the bob of a pendulum.*

The form of the associated wave for longitudinal pendulum-vibrations is shown in Fig. 17 *bis*.

Fig. **17** *bis.*
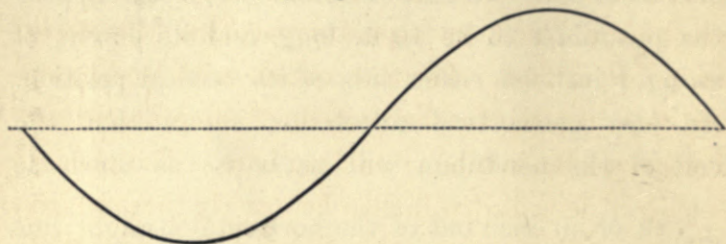
Retaining the *form* of the curve, we may make its amplitude and wave-length as large or as small as we please, as in the case of the waves in Fig. 4, (1) and (2), p. 13.

20. We have examined the transmission of waves due to longitudinal vibrations along a single very slender filament. Suppose that a great number of such filaments are placed side by side in contact with each other, so as to form a uniform material column. If, now, precisely equal waves are simultaneously transmitted along all the constituent filaments, successive pulses of condensation and rarefaction will pass along the column. The parts in any assigned transverse section of the column will, obviously, at any given moment of time, all have exactly the same degree of compression or dilatation.

When a pulse of condensation is traversing the section, its parts will be more dense, and when a pulse of rarefaction is traversing it, less dense, than they would be were the column transmitting no waves at all, and its separate particles, therefore, absolutely at rest.   Let the column with which we have been dealing be the portion of atmospheric air enclosed within a tube of uniform bore.   The phenomena just described will then be exactly those which accompany the passage of a sound from one end of the tube to the other.   It remains to examine the mechanical cause to which these phenomena are due.

Atmospheric air in its ordinary condition exerts a certain pressure on all objects in contact with it. This pressure is adequate to support a vertical column of mercury about 30 inches high, as we know by the common barometer.   Fig. 18 represents a section of a tube closed at one end, with a movable piston fitting air-tight into the other.   In (0) the air on both sides of the piston is in the ordinary atmospheric condition, so that the pressure on the right face of the piston is counteracted by an exactly equal and opposite pressure on its left face.

In (1) the piston has been moved inwards, so as to compress the air on the right of it.   That on its left, being in free communication with the external

air, is not permanently affected by the motion of the piston. In order to retain the piston in its forward position, it is necessary to *exert a force upon it* in

Fig. 18.

(0)

(1)

(2)
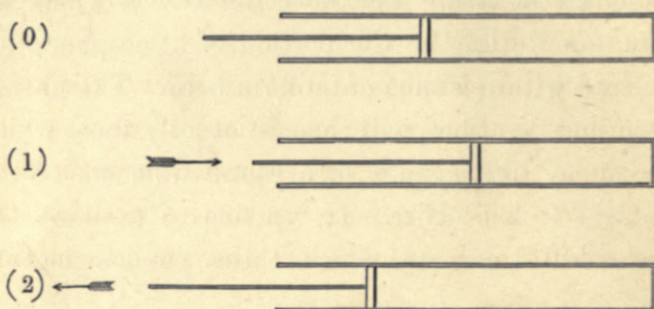
the direction of the arrow. If this force be relaxed, the piston will be driven back. Since the pressure of the air on the left of the piston is just what it was before, that on its right must necessarily have increased. But this increase of pressure is accompanied by an increase of density due to the compression of the air on the right of the piston. Hence *increase of pressure accompanies increase of density.* If, as in (2), we reverse the process by moving the piston outwards, the extraneous force must be exerted in the opposite direction shown by the arrow. The pressure on the right of the piston is therefore less than the normal atmospheric pressure on its left,

i.e. diminution of pressure accompanies diminution of density. By experiments such as the above it was shown, by the French philosopher *Mariotte*, that the pressure of air *varies as its density.*

21.   Next, let us take a cylindrical tube open at one end and having a movable piston fitting into the other, as in Fig. 19.

In (0) the piston is at rest at *A*, and the air in its ordinary atmospheric condition of density and pressure.   In (1) the piston is pushed inwards as far as *C*.   While it is moving up to this position, the air-particles in front of it are thrown into motion.

Fig. 19.
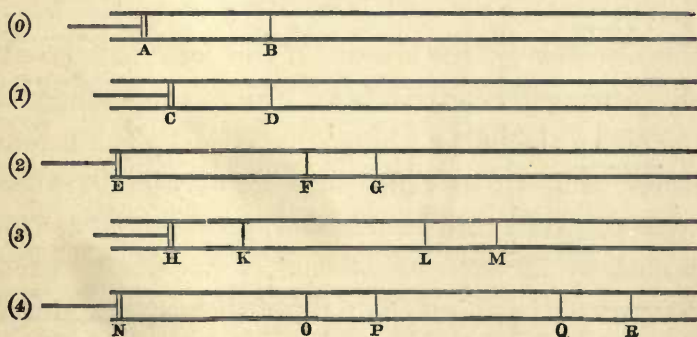
Suppose that, at the moment when the piston reaches *C*, the particles at *D* are just beginning to be disturbed.   The air which, in (0), occupied *AB* is now crowded into *CD*, and is therefore denser than that further on in the tube.   Next, let the piston be

drawn back to *E*, (2), as much to the left of its
original position, *A*, (0), as in (1) it was to the
right of it. The air in *CD*, (1), will, while this is
taking place, expand into *EF*; for, being denser, it
will also be at a greater pressure than the air to the
right of it. It will, therefore, act on this air in the
same way as the piston did on the air in contact
with it when moving from *A*, (0), to *C*, (1). Hence
the air in *FG* will be condensed, *G* being the point
where the air particles are just beginning to be dis-
turbed at the moment when the piston reaches the
position *E*. Thus the air at *D* advances to *F*.

Fig. 19.
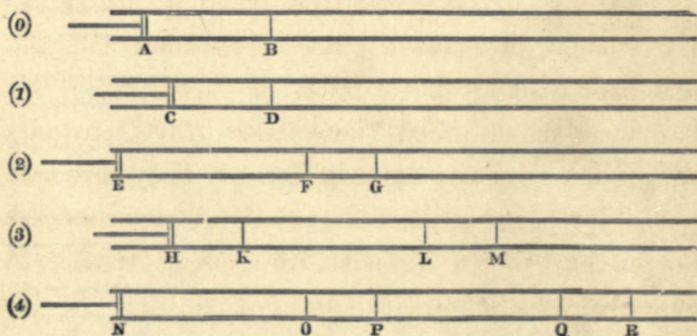
Further, in consequence of the backward motion of
the piston, the air in the neighbourhood of *C*, (1),
moves up to *E*, (2). Thus the air originally in
AB now occupies *EF*, which is greater than *AB*.
It is therefore less dense than in (0), *i.e.* is in a

state of rarefaction. Now, let the piston again advance to H, (3). The air in FG, being at a greater pressure than that in its front and rear, will expand in both directions, causing a new condensation, LM, to be formed further on, and itself becoming the rarefaction KL, co-operating at the same time with the advancing piston to produce in its own rear the condensation HK. In (4) the piston is again where it was in (2). HK has expanded into the rarefaction NO, KL contracted into the condensation OP, LM expanded into the rarefaction PQ, and a new condensation, QR, been formed in front.

The figure makes it clear that each forward stroke of the piston produces a pulse of condensation, and each backward stroke a pulse of rarefaction; but that, when once formed, these pulses travel onwards independently of any external force. They do so, as we have seen, in virtue of the relation which connects the pressure of the air with its density, that is to say, on the *elasticity* of the air.

If we suppose our movable piston withdrawn from the tube, and a vibrating tuning-fork held with the extremity of one prong close to the orifice of the tube, the conditions of the problem will not be essentially modified. Each outward swing of the prong will give rise to a condensed, and each inward

swing to a rarefied pulse, and thus, during every complete vibration of the fork one sonorous wave, consisting of a pulse of condensation and a pulse of rarefaction, will be started on its journey along the tube.

22. We have examined the transmission of Sound along a column of air contained in a tube of uniform bore. A more important case is that in which a sound originated at an assigned point spreads out from it freely in all directions. Here we must conceive a series of spherical shells, alternately of condensed and of rarefied air, one inside the other, and having the point of origination of the sound as their common centre. All the shells must be supposed to expand uniformly like an elastic globular balloon constantly inflated with more and more gas. Of course no shell in any two consecutive positions consists of exactly the same materials. The condensations and rarefactions spread progressively: the air-particles merely vibrating about their positions of original rest. The great difference between this case and that last considered lies in the fact, that, as the spherical shells of condensation and rarefaction spread, it is necessary, in order to keep up the wave-motion, to throw larger and larger surfaces of air into vibration; whereas within the tube the transverse section remained the same throughout. Hence, as the same amount of

original disturbing force has to set a constantly in-
creasing number of air-particles into motion, it can
only do so by correspondingly shortening the distances
through which the individual particles move, *i.e.* by
diminishing their extent of vibration.   Accordingly
when Sound-waves spread out freely in all directions,
the further any given air-particle is from the point
at which the sound originated, the smaller will be
the extent of the vibration into which it will be
thrown when the waves reach it.

23.   Sounds are either *musical* or *non-musical*.
The vast majority of those ordinarily heard—the
roaring of the wind, the din of traffic in a crowded
thoroughfare—belong to the second class.   Musical
sounds are, for the most part, to be heard only from
instruments constructed to produce them.   The dif-
ference between the sensations caused in our ears by
these two classes of sounds is extremely well marked,
and its nature admits of easy analysis.   Let a note
be struck and held down on the harmonium, or on
any instrument capable of producing a sustained
tone.   However attentively we may listen, we per-
ceive no change or variation in the sound heard.
A perfectly continuous and uniform sensation is
experienced as long as the note is held down.   If
instead of the harmonium we employ the pianoforte,
where the sound is loudest directly after the moment

of percussion, and then gradually dies away, diminution of loudness is the only change which occurs.

In the case of non-musical sounds variations of a different kind can be easily detected. In the howling of the wind the sound rises to a considerable degree of shrillness, then falls, then rises again, and so on. On parts of the coast where a shingly beach of considerable extent slopes down to the sea, a sound is heard in stormy weather which varies from the deep thundering roar of the great breakers, to the shrill tearing scream of the shingle dragged along by the retreating surf. Similar variations may be noticed in sounds of small intensity such as the rustling of leaves, the chirping of insects, and the like. The difference, then, between musical and non-musical sounds seems to lie in this, that the former are constant, while the latter are continually varying. The human voice can produce sounds of both classes. In singing a sustained note it remains quite steady, neither rising nor falling. Its conversational tone, on the other hand, is perpetually varying in height even within a single syllable; directly it ceases so to vary, its non-musical character disappears, and it becomes what is commonly called 'sing-song.'

We may then define a musical sound as a *steady* sound, a non-musical sound as an *unsteady* sound. It is true we may often be puzzled to say whether

a particular sound is musical or not : this arises, however, from no defect in our definition, but from the fact that such sounds consist of two elements, musical and non-musical, of which the latter may be the more powerful, and therefore absorb our attention until it is specially directed to the former. For instance, a beginner on the violin often produces a sound in which the irregular scratching of the bow predominates over the regular tone of the string. In bad flute-playing an unsteady hissing sound accompanies the naturally sweet tone of the instrument, and may easily surpass it in intensity. In the tones of the more imperfect musical instruments, such as drums and cymbals, the non-musical element is very prominent, while in such sounds as the hammering of metals, or the roar of a water-fall, we may be able to recognise only a *trace* of the musical element, all but extinguished by its boisterous companion.

We have seen that Sound reaches our ears by means of vibrations executed by the particles of the atmosphere. It has also been shown that *steadiness* is the characteristic feature of musical, as distinguished from non-musical, sounds. We may infer hence that the motion of the air corresponding to a single musical sound will be itself steady, *i.e.* that *equal numbers of equal vibrations will be executed in*

precisely equal times. This conception of the physical
conditions under which musical sounds are produced
will suffice for the present. We proceed to consider
in detail the various ways in which such sounds
may differ from each other, and to investigate the
mechanical cause to which each such difference is to
be referred. In what follows, by the word 'sound'
will always be meant 'musical sound,' unless the
contrary be expressly stated.

CHAPTER II.

24. A musical sound may vary in three different respects. Let a note be played, first by a single violin, then, by two, by three, and so on, until we have all the violins of an orchestra in unison upon it. This is a variation of *loudness* only. Next let a succession of notes be played on any instrument of uniform power, such as the harmonium without the expression-stop, or on the principal manual of an organ, only one combination of stops being in either case used. Here we have a variation of *pitch* only. Lastly, let one and the same note be successively struck on a number of pianofortes of the same size, but by different makers. The sounds heard will all have exactly the same pitch, and *about* the same degree of loudness; nevertheless they will exhibit decided differences of character. The tone of one instrument will be rich and full, of another ringing and metallic, that of a third will be described as 'wiry,' of a fourth as 'tinkling,' and so on.

Sounds thus related to each other are said to vary in *quality* only. The instances just considered

4—2

have the advantage of simplicity, since they allow of changes in loudness, pitch and quality being exhibited separately.   They are, however, less strik-ing than other cases where sounds vary in two, or in all three, of these respects at the same time.   A practised ear may be requisite to detect the difference between the tones of two pianofortes, but no one is in danger of mistaking, for instance, a flute for a trumpet.   There is here, no doubt, considerable differ-ence of loudness as well as of quality, but let the more powerful instrument be placed at such a dis-tance that it sounds no louder than the weaker one, and the distinction between the two kinds of *tone* will be still quite decisive.

Two assigned musical sounds may differ from each other in loudness *or* pitch *or* quality, and agree in the other two—or they may differ in any two of these, and agree in the third—or they may differ in all three.   There is, however, no *other* respect in which they *can* differ, and accordingly we know all about a musical sound as soon as we know its loudness, its pitch and its quality.   These three elements *determine* the sound, just as the lengths of the three sides of a triangle determine the triangle.

25.   The loudness of a musical sound depends entirely, as we shall easily see, on the extent of vibratory movement performed by individual par-

ticles composing the conveying medium. A sound-producing instrument can be readily observed to be in a state of rapid tremor. The vibrations of a tuning-fork are recognisable by the eye in the fuzzy, half-transparent, rim which surrounds its prongs after it has been struck ; and by the touch, if we place a finger gently against one of the prongs. The harder we hit the fork the louder is its sound, and the larger, estimated by both the above modes of observation, are its vibrations. The experiment may be tried equally well on any pianoforte whose construction allows the wires to be uncovered. It is natural to infer that a vibration on the part of a sound-producing instrument communicates to the particles of the air in contact with it a corresponding movement. Thus a sound of given loudness is conveyed by vibrations of given extent, and, if the sound increases or diminishes in intensity, the extent of the vibrations which carry it must increase or diminish correspondingly.

We conclude, then, that the loudness of a musical sound depends solely on the extent of excursion of the particles which constitute the conveying medium *in the neighbourhood of our ears.* This limitation is clearly essential, since a sound grows more and more feeble the greater our distance from the point where it is produced. This diminution of intensity

as the distance from the origin of sound increases
is a direct consequence of the connexion between
loudness and extent of vibration. We have seen
[§ 22] that the further an air-particle is from the
point where a sound is produced, the smaller will be
the extent of the vibration into which it is thrown by
the sonorous wave. Hence, as the sound advances it
will necessarily become feebler, provided always that
the waves are permitted to spread out in all direc-
tions. If they are confined, say, in a tube, the
intensity of the sound will for considerable distances
remain practically constant. We have here the
theory of the message-pipes which are used in large
establishments to enable a conversation to be carried
on between distant parts of a building. A whisper,
inaudible to a person close to the speaker, may by
their means be perfectly well heard by a listener at
the other end of the tube.

26.   We have next to enquire to what mechani-
cal causes differences in the *pitch* of musical sounds
are to be referred. Rough observation at once in-
dicates the direction in which we must look. If we
draw the point of a pencil along a rough surface,
first slowly and then more quickly, the sound heard
will be distinctly shriller the more rapid the move-
ment of the pencil. As its point passes over the
minute elevations and depressions which constitute

the roughness of the surface, a series of irregular vibrations are set up in the materials of the surface, and of the pencil, and by them communicated to the air. The more rapid these vibrations, the shriller is the sound produced. The instrument described below, which is called a 'Syren,' gives us the means of following up with accuracy the hint just obtained.

Fig.20
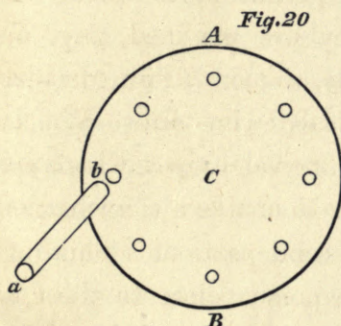
AB is a thin circular disc of metal or card-board, which by means of a multiplying wheel, can be set in rapid uniform rotation in its own plane about a fixed axis through its centre, C. A series of holes (eight in the figure) are punched in the disc at equal distances along a circle having its centre at C. A small tube, ab, is held with one end close to one of the holes. If, while the disc is rotating, we blow steadily and continuously into the tube at a, a certain quantity of air will pass through the disc

whenever a hole passes across the orifice b of the tube ab. During the intervals of time which elapse between the passage of adjacent holes across b, *no* air can pass through the disc. Hence, if the disc be rotating uniformly, a series of such discharges will succeed each other at precisely equal intervals of time. The air on the other side of the disc will necessarily be agitated by the process. Every time that air is driven through one of the holes, an increase of pressure occurs close to it, and accordingly a pulse of condensation is formed there. The elastic force of the air will give rise to a pulse of rarefaction during each interval between successive discharges. Hence the Syren supplies us with a regular series of the alternate condensations and rarefactions which, as we have seen, constitute waves of Sound.

27. While air is being blown steadily into the tube, let the disc be made to rotate slowly, and then with gradually increasing rapidity. At first nothing will be audible but a series of faint intermittent throbs, due to the impact of the air driven through the tube against the successive portions of the disc which separate its holes. This sound may be exactly reproduced by moving the fore-finger to and fro rapidly before the lips, while blowing through them. It contributes nothing to the proper musical sound of the instrument, and is only audible in its imme-

diate neighbourhood.   Presently a deep musical
sound begins to be heard, which, as the velocity of
rotation increases, constantly rises in pitch.   The
acuteness of the sound thus obtainable depends
solely on the speed to which we can urge the instru-
ment, and is therefore limited only by the driving-
power at our command.   The rise of pitch in this
experiment is perfectly *continuous*, that is to say,
the sound of the Syren, in passing from a graver
to a more acute note, goes through *every possible
intermediate degree of pitch*.   It is important that
we should familiarise ourselves with this conception
of the continuity of the scale of pitch, because in
the instrument from which our ideas on this subject
are usually obtained—the pianoforte—the pitch
alters *discontinuously, i.e.* by a series of jumps of
half a Tone each, and we are thus tempted to ignore
the intervening degrees of pitch, or even to suppose
them non-existent.   The more perfect musical in-
struments, such as the human voice or the violin,
are as capable as the Syren of passing through all
degrees of pitch from one note to another in the
way called '*portamento*' or '*slurring.*'

It is clear from the nature of the Syren's con-
struction that the only change which can take place
during the rise in pitch is the increased number of
impulses communicated to, and therefore of vibrations

set up in, the external air, during any given interval
of time. If, when a note of given pitch has been
attained by the Syren, we check any further increase
of velocity and cause the disc to rotate uniformly
at the rate which it has just reached, no further
alteration of pitch will occur, and the note will be
steadily held by the instrument so long as the uni-
form rotation of its disc is kept up. Hence *the num-
ber of aerial vibrations executed in a given time de-
termines the pitch of the sound heard.*

28. The Syren, besides teaching us this most
important fact, gives us the means of determining
the number of vibrations corresponding to any given
note. If we know the number of rotations which
the disc has performed in a given time, we have only
to multiply this number by the number of holes on
the disc in order to ascertain how many tube-dis-
charges have occurred, and therefore how many
corresponding aerial vibrations have been performed,
in the period in question. The Syren is provided
with a counting-apparatus which registers the num-
ber of times its disc rotates per second.

In order, therefore, to obtain the number of vibra-
tions in a second which correspond to an assigned
note, we have only to proceed as follows. Let the
note be steadily sounded by some instrument of
sustained power, *e.g.* organ or harmonium, and then

cause the Syren-sound to mount the scale until its pitch coincides with that of the note under examination. During coincidence ascertain by the counting-apparatus the number of rotations which are being performed per second. This, multiplied by the number of holes in the disc, gives the number of vibrations per second required. It will be convenient, for shortness' sake, to call the number of vibrations per second to which any note is due the *vibration-number* of the note in question. It is clear, from what has gone before, that any assigned degree of pitch can be permanently registered when once its vibration-number has been ascertained.

29. The Syren shows that below a certain rate of vibration no musical sounds are produced. The position of the absolute limit thus placed to the gravity of such sounds cannot be exactly defined, and probably varies somewhat for different ears. The lowest note on the largest modern organs has $16\frac{1}{2}$ for its vibration-number, but it is a moot question whether the musical character of this note can be recognised.

In any case we may regard the lower limit of musical sounds as situated in the immediate neighbourhood of this degree of pitch. For some distance above the limit the musical character continues very imperfect, and it is not until we reach about 40

vibrations per second that we get a satisfactory musical sound. The limit which bars the scale of pitch in the opposite direction varies greatly in position with different persons; some hearing with the utmost distinctness a shrill sound which others declare to have for them no existence. A cleverly-planned little instrument called *Galton's whistle,* sold by philosophical instrument makers, the note of which can be made to vary at pleasure within certain limits, brings out these differences of individual perception in a striking manner.

Sounds well within the limit of general audibility are, if above a certain degree of acuteness, painful to the ear, and therefore unfit for musical purposes. The highest note of the piccolo, the shrillest sound heard in the orchestra, makes 4,752 vibrations per second*. This we may regard as constituting a practical superior limit to the scale of pitch at the disposal of musical art. The extreme range attainable by exceptional human voices, from the deepest note of a bass to the highest of a soprano, lies, roughly speaking, between 50 and 1,500 vibrations per second. Ordinary chorus voices range from 100 to 900 or 1,000 vibrations per second.

The power of discriminating minute differences of pitch varies greatly with the acuteness of percep-

* Taking 264 vibrations for middle-C (German pitch).

tion possessed by individual observers.   An experi-
ment which a well-known Cambridge tuner, Mr
James Ling, was kind enough to make in my presence
showed that in the neighbourhood of middle-C of the
pianoforte he was able to distinguish, by listening to
them *successively*, two sounds forming with each
other an interval so small that it would take about
250 such intervals to make up a single octave.   The
difficulty of discriminating pitch increases rapidly
with very low or very high sounds ; the above
instance of what can be achieved by the most
highly-trained ears applies, therefore, only to a few
octaves in the middle region of the scale.   The
continuously shading-off gradations presented by
the scale of pitch afford no antecedent presumption
that particular sets of sounds should be singled out
to form the materials of Music.   Nevertheless it
is found that practically the group of different
sounds employed in a musical composition is, re-
latively to the whole available stock, extraordinarily
limited.

30.   When one sound has been arbitrarily selected
as the starting-point, there are certain other sounds,
having fixed relations of pitch to that previously
chosen, which are capable of forming, with it and
with each other, melodic and harmonic effects es-
pecially pleasing to our ears.   These are the notes

of the ordinary Major and Minor scales, the original sound of reference being the common *tonic**, or *key-note*, of those scales. In saying that these sounds have fixed mutual relations of pitch, we merely state formally an obvious fact. A familiar melody is recognised equally well whether heard in the deep tones of a man's, or in the shrill notes of a child's voice. Whether the singer 'pitches' it on a low or on a high note of his voice makes no difference in the melody itself. In fact the correctness with which an air is sung no more depends on the exact pitch of the note on which the singer starts it, than does the faithfulness of a plan on the precise *scale* which the draughtsman has adopted. It is sufficient that the constituent notes of the melody should have fixed mutual *relations* of pitch, just as in the plan, the several objects represented need only be drawn *in proportion* to their actual dimensions.

The difference in pitch of any two notes is called the *interval* between them: it is on accuracy of intervals that Music essentially depends.

31. The most important interval in the scale is the *Octave*. It is that which separates the highest note of a peal of eight bells from the lowest. When

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* The tonic may be recognised by its being the note on which a tune almost invariably ends, and on which it *must* end if an impression of entire completeness is to be conveyed to the hearer.

a bass and a treble voice sing the same melody together, the notes of the latter are usually an Octave above those of the former. This interval has the property, shared by no other, that if, starting from any note we choose, we ascend to that an Octave above it, then to that an Octave above the last, and so on, we get a number of notes which sound perfectly smooth and agreeable when heard all together. The same thing holds good if we take the succession of Octaves in descending order from the note fixed on as the starting-point. Hence the whole scale of pitch may be conveniently divided into a series of Octaves taken upwards and downwards from some one sound arbitrarily selected. Narrower intervals situated in any one Octave are repeated in all the others, so that when we have settled those intervals for *a single Octave* we have settled them for all the rest. Within the limits of each Octave the Major scale presents us with *seven notes*, or if we include that which forms the starting point of the next cycle, with *eight*. The fact that the eighth note is the Octave of the first explains the meaning of the word 'Octave,' *i.e.* 'eighth' (*Latin:* '*octavus*').

The eight notes are those of an ordinary peal of the same number of bells, or of the white keys of the pianoforte between two adjacent C's. We may for

convenience of reference number them 1, 2, 3......8, beginning with the lowest note which is their tonic. The following nomenclature is used to describe the intervals formed by the several notes *with the tonic.*

Notes forming interval.	Name of interval.
1 and 2	Second
1 3	Major Third
1 4	Fourth
1 5	Fifth
1 6	Major Sixth
1 7	Major Seventh
1 8	Eighth or Octave.

When two notes of the same pitch are sounded together, *e.g.* by two instruments or by two voices, the notes are said to be in *unison.* Though there is here no difference of pitch whatever, it is convenient to rank the unison as an interval. With this explanation we may add to the above table that 1 and 1 form the interval of an unison. The reader must carefully avoid giving to the 'Thirds,' 'Fourths,' &c., which he meets with in Music the meanings attached to the same words in the Arithmetic of fractions, with which they have absolutely nothing to do. A 'Fifth,' for example, does not stand for a *fifth part*

of an Octave, or indeed for a *fifth part* of anything, but for the difference in pitch between the first and fifth notes of the scale.

The several pairs of notes which form the intervals laid down in our table do not all produce smooth and agreeable impressions. The five pairs

1—3, 1—4, 1—5, 1—6, 1—8

yield smooth effects : the remaining two,

1—2 and 1—7,

are decidedly harsh-sounding combinations. The intervals in the first line are therefore classed as *concords*, those in the second as *discords*.

32. The Minor scale has the notes 1, 2, 4, 5, 8 in common with the Major scale. It substitutes for 3 a sound lying between that note and 2 and forming with 1 a consonant interval called the Minor Third. According to circumstances it may either retain 6, or replace it by a sound lying between that note and 5, and forming with 1 a concord called the Minor Sixth. Similarly it may employ 7, or, in the room of that note, a fresh sound situated between it and 6, and forming with 1 a discord called the Minor Seventh.

Thus, if we include the Octave, the two scales together give us a series of eleven notes, which, severally combined with the tonic, form ten distinct

intervals. They are expressed in musical notation
as follows :—

The reader should endeavour to familiarise him-
self with these intervals, so that, when he hears the
pair of notes which form any one of them successively
sounded, he may at once be able to name the interval.

33. The Syren enables us to obtain simple nu-
merical measures of the intervals exhibited on this
page.

Let a second circular row, containing sixteen
holes, be punched in its disc, and the instrument be
set rotating uniformly. If we now blow alternately
against the 8-hole row and the 16-hole row, we shall
find that the sound produced at the latter is precisely
one Octave higher than the sound produced at the
former. If we increase or diminish the velocity of
rotation, both sounds will, of course, rise or fall to-
gether, but the interval between them will remain
unaffected and equal, as before, to an exact Octave.
The number of air-discharges corresponding to the
more acute sound is, in this case, evidently twice as
large in any given time as the number during the

same time for the graver sound.   Accordingly we
have the following result :—

*When two sounds differ by a single Octave, the
higher sound makes exactly twice as many vibrations
in any assigned time as the lower.*

Next let a row of 12 holes be punched in the
disc of the Syren.   Taking this row with the 8-hole
row, and proceeding as in the last instance, we find
that the more acute sound forms a Fifth with the
graver one.   The numbers of discharges in any given
time are here as 12 to 8, *i.e.* as 3 to 2.   The result
therefore is as follows :—

*When two sounds differ by a Fifth, the higher
sound makes exactly three vibrations during the time
in which the lower sound makes two.*

If we take the 16-hole and 12-hole rows together,
the interval amounts to a Fourth; accordingly, *when
two sounds differ by a Fourth, the higher sound makes
exactly four vibrations during the time in which the
lower sound makes three.*

34.   The results just obtained admit of being
somewhat more concisely stated.   During *one second
of time* the upper of two sounds differing by an Octave
makes a number of vibrations which is to the number
made by the lower sound as 2 to 1.   For a Fifth the
ratio is 3 to 2.   For a Fourth it is 4 to 3.   Remem-
bering, then, the definition of the vibration-number

of an assigned sound [§ 28], we may express our three results as follows:—

When two sounds form with each other the intervals of an Octave, a Fifth or a Fourth, their vibration-numbers are to each other, in the first case as 2 to 1, in the second as 3 to 2, in the third as 4 to 3.

A ratio is most easily expressed by a fraction. Thus we may regard the fraction $\frac{3}{2}$ as denoting the interval of a Fifth. It may be taken as an abbreviated statement of the fact that, when two sounds form a Fifth with each other, the more acute makes 3 vibrations while the graver makes 2.

35. By suitable experiments, similar numerical relations to those already established may be obtained for all the intervals already considered. A fraction can thus be determined for each interval, in the manner exemplified in the case of the Fifth. We will call this fraction the *vibration-fraction* of the interval in question. The accompanying table gives, in the second column, the vibration-fractions corresponding to the intervals named in the first; and in the third, states the consonant or dissonant character of each interval.

It is noticeable that the dissonant intervals involve higher numbers in their vibration-fractions than the consonant intervals do; the latter, with the

solitary exception of the Minor Sixth, having no-
thing beyond 6, while the former bring in 9, 15
and 16.

Name of interval.	Vibration-fraction.	Character of interval.
Unison	$\frac{1}{1}$	concord
Second	$\frac{9}{8}$	discord
Minor Third	$\frac{6}{5}$	concord
Major Third	$\frac{5}{4}$	concord
Fourth	$\frac{4}{3}$	concord
Fifth	$\frac{3}{2}$	concord
Minor Sixth	$\frac{8}{5}$	concord
Major Sixth	$\frac{5}{3}$	concord
Minor Seventh	$\frac{16}{9}$	discord
Major Seventh	$\frac{15}{8}$	discord
Octave	$\frac{2}{1}$	concord

36. By the help of this table we can calculate
the vibration-numbers of all the notes within a
single Octave which belong to the Major or Minor
keys *as soon as the vibration-number of their tonic is
given.* For instance, let middle C of the pianoforte
(vib.-no. 264) be the tonic. From the third line
of the table we see that the vibration-number of
D must be to 264 in the ratio of 9 to 8. It must
therefore be equal to $\frac{9}{8} \times 264$, or 297. For E♭, by

exactly similar reasoning, we obtain $\frac{6}{5} \times 264$ or $316\frac{4}{5}$;

for E, $\frac{5}{4} \times 264$, or 330.

The student should work out the remaining cases for himself.

The complete results for the Major scale are as follows :—

| 264 | 297 | 330 | 352 | 396 | 440 | 495 | 528 |
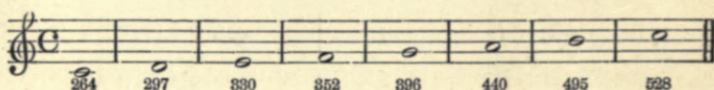
In order to extend the scale an Octave upwards we have only to multiply each vibration-number by 2. A second multiplication by 2 will raise it by another Octave, and so on. Conversely, in order to pass to the Octave below we divide each vibration-number by 2. To descend a second Octave we repeat the operation, and so on.

Thus the pitch of the tonic absolutely fixes the pitch of every note in the scale of which it is the starting-point.

Before proceeding to investigate the mechanical equivalent of the third element [§ 24] of a musical sound, its quality, we will briefly examine, in the next chapter, a subject possessing an important bearing on that enquiry.

CHAPTER III.

37. When a sounding body causes another body to emit sound, we have an instance of a very remarkable phenomenon called *resonance*. The German term for it, ' co-vibration ' (*Mitschwingung*), possesses the merit of at once indicating its essential meaning, namely the setting up of vibrations in an instrument, not by a blow or other immediate action upon it, but indirectly as the result of the vibrations of another instrument. In order to produce the effect, we have only to press down very gently one of the keys of a pianoforte, so as to raise the damper without making any sound, and then sing the corresponding note loudly into the instrument. When the voice ceases, the instrument will continue to sustain the note, which will then gradually fade away. If the key is allowed to rise again before the sound is extinct, it will abruptly cease. A similar experiment may be tried, as follows, on any pianoforte which allows the wires to be uncovered. Each note is, as is well known,

produced by two or by three wires. Having, as in the previous case, raised one of the dampers without striking the note, twitch *one* of the corresponding wires sharply with the finger-nail, and then wait a few seconds. The vibrations will, during this interval, have communicated themselves to the other string or strings belonging to the note pressed down : if, now, the first wire be stopped by applying the tip of the finger to the point where it was at first twitched, the same note, due to transmitted vibrations, will continue to be sustained by the remaining wire or wires.

A more instructive method of studying resonance is to take two unison tuning-forks, strike one of them, and hold it a short distance from the other. The second fork will then commence sounding by resonance, and will continue to produce its note though the first fork be brought to silence. It is essential to the success of this experiment that the two forks should be rigorously in unison. If the pitch of one of them be lowered by causing a small pellet of wax to adhere to the end of one of its prongs, the effect of resonance will no longer be produced, even though the alteration of pitch be too small to be recognised by the ear. Further, the phenomenon generally requires a certain length of time to develope itself; for, if the silent fork be only *momentarily*

exposed to the influence of its sounding fellow, hardly any appreciable result ensues. The resonance, when produced, is at first extremely feeble, and gradually increases in intensity under the continued action of the originally-excited fork. Some seconds must elapse before the maximum resonance is attained. The conditions of our experiment show that the resonance of the second fork was due to the transmission *by the air* of the vibrations of the first, the successive air-impulses falling in such a manner on the fork as to produce a *cumulative effect*. If we bear in mind the disproportionate mass of the body set in motion compared to that of the air acting upon it,—steel being more than six thousand times as heavy as atmospheric air for equal bulks— we cannot fail to regard this as a very surprising fact.

Let us examine the mechanical causes to which it is due. Suppose a heavy weight to be suspended from a fixed support by a flexible string, so as to form a pendulum of the simplest kind. In order to cause it to perform oscillations of considerable extent by the application of a number of small impulses, we proceed as follows. As soon as, by the first impulse, the weight has been set vibrating through a small distance, we take care that every succeeding impulse is impressed *in the direction in which the weight is moving at the time.* Each impulse thus applied will

cause the pendulum to oscillate through a larger angle [§ 19] than before, and, the effects of many impulses being in this way added together, an extensive swing of the pendulum is the result.

When the distance through which the weight travels to and fro, though in itself considerable, is *small compared to the length of the supporting string,* the time of oscillation is the same for any extent of swing within this limit, and depends only on the length of the string. My readers will find this important principle illustrated in any manual of Elementary Mechanics, and I must ask them to take it for granted here. For the sake of simplicity, let us suppose that we are dealing with a pendulum of such a length as to perform one *complete* oscillation in each second, and therefore to make a single forward or backward swing in each half second. It will be clear, from what has been said above, that the maximum effect will be produced on the motion of the pendulum by applying a forward and a backward impulse respectively during each alternate half second, or which is the same thing, administering *a pair of to-and-fro impulses during each complete oscillation of the pendulum.* We have a simple instance of such a proceeding in the way in which two boys set a heavily-laden swing in violent motion. They stand facing each other, and each boy, when the swing is

moving away from him, helps it along with a vigorous push.

38.   The above considerations enable us to explain how a sounding fork can set another fork in unison with itself into vibration through the action of the intervening air.   When a continuous musical note is being transmitted, we know that, at any one point we choose to fix upon, the air is undergoing a series of rapid changes, becoming alternately denser, and less dense than it would be but for the passage of the sound.   The increase of density is accompanied by an increase of pressure ; its diminution by a diminution of pressure [§ 20].

Fig. 21
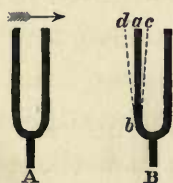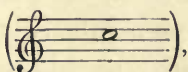
Let *A*, Fig. 21, be the sounding fork, *B* that whose vibrations are to be excited by resonance, and let us consider the effect of the alternations of pressure of the air at *a* on the prong *ba*.   Let the first pulse which arrives at *a* be one of condensation and let its increased pressure cause the prong *ba* to swing as far as *bc**.   The elastic recoil of the fork would

* The extents of vibration shown in this figure are of necessity enormously exaggerated.

alone cause the prong to spring back to an equally
inclined position on the other side of *ba*. Since,
however, the period of vibration is by supposition
the same for the two forks, a pulse of rarefaction,
entailing diminished pressure, will arrive at *a* just as
the prong is passing through its erect position *ba*.
The prong will therefore swing a little further to
the left than it would otherwise have done, say to
bd, making the angle *abd* slightly larger than the
angle *abc*. The prong then starts to the right and
is, while passing through the position *ba*, again helped
on by the arrival there of a new pulse of condensa-
tion. Thus the alternate condensations and rare-
factions of the Sound-waves play towards the fork
exactly the parts sustained by the two boys in our
illustration of the swing. Accordingly, these air-
impulses are applied under precisely the conditions
which we saw to be most favourable to the rapid
development of vibratory motion. Their large num-
ber makes up for the feebleness of each separate
impulse. Accordingly resonance is produced more
slowly between unison-forks of low than between
those of high pitch. I find that, with two forks
making 256 vibrations per second, about one second
is requisite to bring out an audible resonance ; while
with another pair, making 1,920 vibrations per second,
I can with difficulty damp the first fork sufficiently

soon after striking it to prevent the other from making itself heard.
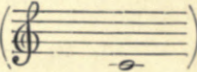
39. A column of air is easily set in resonant vibration by a note of suitable pitch. The roughest experiment suffices to establish this fact. We have only to roll up a piece of paper, so as to make a little cylinder twelve inches long and an inch or two in diameter, with both ends open, strike a common C tuning-fork, (♭𝄞 𝅝), and hold it close to one of the apertures. As soon as the fork reaches the position (1) Fig. 22,* its tone will unmistakably swell out. In order to estimate the increase of intensity produced, it is a good plan to move the fork rapidly to and fro, a few times, between the positions (1) and (2).

Fig. 22.
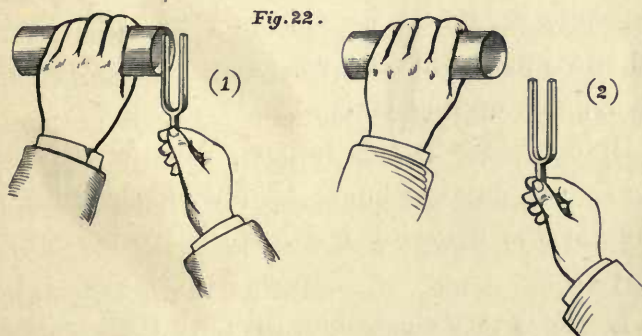
In the first case we have the full effect of resonance,

* This figure was drawn for a cylinder only six inches in length, but suffices for the purpose of illustration for which it is here used.

in the second only the unassisted tone of the fork, and the contrast is very marked. We may shorten or lengthen our cylinder within certain limits and still obtain the phenomena of resonance, but the greatest reinforcement of tone we can attain with the fork selected will be produced by an air-column about twelve inches long.

If we close one end of the paper cylinder, by placing it, for instance, on a table, and repeat our experiment at the open end, only a very weak resonance is produced; but we obtain a powerful resonance by operating with a fork making *half as many vibrations per second* as that before employed. In this case, then, a column of air contained in a cylinder of which one end was closed resounded powerfully to a note an Octave below that which elicited its most vigorous resonance when contained in a cylinder open at both ends.

By operating in this fashion, with forks of different pitch, on air-columns of different lengths, we arrive at the following laws, which are universally true :—

1. For every single musical note there exists a corresponding air-column of definite length which resounds the most powerfully to that note.

2. The maximum resonance of air in a closed

pipe is produced by a note one Octave below that to which an open pipe of the same length resounds the most powerfully.

40.   In order to ascertain the precise relation between the pitch of a note and the length of the corresponding air-column, we will examine the way in which resonance is produced in a column of air contained in a pipe closed at one end.

Let A, Fig. 23, be the open, and B the closed, ends of the pipe, and let us for a moment replace the contained air by an elastic spiral spring fastened at B, and of length equal to AB.

Fig. 23.

B   A
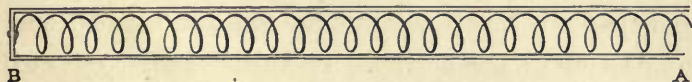
Suppose the end of the spring suddenly pushed a little way from A towards B.   The coils of the spring nearest A will be squeezed together, and this condensed state of the spring will be transmitted to B and, after reflexion there, will travel back to A.   In virtue of the elasticity of the spring its free end will now be caused to protrude beyond the mouth of the pipe as far as it was at first pushed into it.   The coils near A being thus drawn some- what more apart than was the case when they were in their undisturbed condition, a rarefied state of the spring will be produced at A, transmitted to B,

and there reflected. When this rarefaction returns to *A*, the free end of the spring will momentarily resume the position within the mouth of the pipe to which it was originally pushed. The elasticity of the spring thus causes it to *lengthen and shorten as a whole* in consequence of the single push originally given it, and this motion would for a time continue, its successive periods being four times the space of time occupied by a pulse of condensation or rarefaction in traversing the length of the pipe. The free end of the spring may, however, be pushed and pulled alternately so as to reinforce *each* pulse as it arrives at the mouth of the pipe, and in this manner the maximum of motion will be communicated to the spring. In this case, one outward and one inward impulse must be communicated to the free end of the spring during the time which elapses while a pulse traverses four times the length of the pipe. Reverting to the actual conditions of our problem, we have the resonance of the air-column in place of the alternate lengthening and shortening of the spring. The to-and-fro impulses at *A* are impressed by a vibrating fork. The Sound-pulse traverses four times the length of the pipe while the fork is performing one complete vibration. We know, however [§ 15 p. 32], that during this latter period the Sound-pulse produced by the

fork's action traverses precisely one wave-length corresponding to the pitch of the note produced by the fork.   Hence, for maximum resonance in the case of a stopped pipe, the wave-length corresponding to the note sounded must be four times as great as the length of the air-column, or the length of the column one quarter of the wave-length.

41.   These principles give us the explanation of a useful appliance for intensifying the sound of a tuning-fork.   Such a fork, when held in the hand after being struck, communicates but little of its vibrations to the surrounding air ; when, however, its handle is screwed into one side of a wooden box of suitable dimensions, in the way shown in Fig. 24, the tone becomes much louder.   The vibrations of the fork pass from its handle to the wood of the box,

Fig. 24
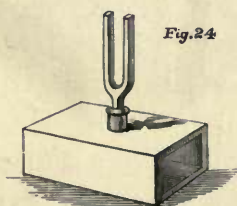
and thence to the air-column within, which is of appropriate length for maximum resonance to the fork's note.   This convenient adjunct to a tuning-fork goes by the name of a 'resonance-box.'

42.   When a number of musical sounds are

simultaneously sustained it is generally difficult, and often impossible, for the unaided ear to decide whether an individual note is, or is not, present in the mass of sound heard. If, however, we had an instrument which intensified the note of which we were in search, without similarly reinforcing others which there was any risk of our mistaking for it, our power of recognising the note in question would be proportionately increased. Such an instrument has been invented by Helmholtz. It consists of a hollow ball of brass with two apertures at opposite ends of a diameter, as shown in Fig. 25.

Fig. 25.
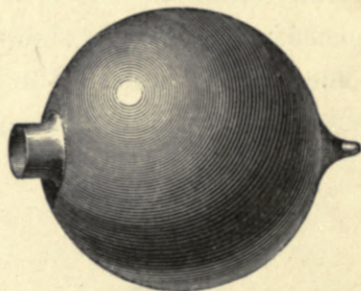
The larger aperture allows the vibrations of the external air to be communicated to that within the ball; the smaller aperture terminates in a nipple of convenient form for insertion in the ear of the observer. The air contained in the ball resounds very powerfully to one single note of definite pitch, whence the instrument has been named by its in-

ventor a *resonator*. The best way of using it is,
first to stop one ear closely, and then to insert the
nipple of the instrument in the other. As often as the
resonator's own note is sounded in the external air,
the instrument will sing it into the ear of the ob-
server with extraordinary emphasis, and thus at once
enable him to single out that note from among a
crowd of others differing from it in pitch. A series
of such resonators, tuned to particular previously
selected notes, constitutes an invaluable apparatus
for analysing a composite sound into the simple
tones of which it is made up.

CHAPTER IV.

ON QUALITY.

43.   The laws of resonance enable us to establish a remarkable, and by most persons utterly unsuspected fact, viz. that the notes of nearly every regular musical instrument with which we are familiar, are not, as they are ordinarily taken to be, single tones of one determinate pitch, but *composite* sounds containing an *assemblage* of such tones. These are always members of a regular series, forming with each other fixed intervals which may be thus stated : if we number the separate single tones of which any given sound is made up, 1, 2, 3, &c., beginning with the lowest, we have

(1)   The deepest, or 'fundamental,' tone, whose pitch is ordinarily regarded as that of the whole sound.

(2)   A tone one Octave above (1).

(3)   A tone a Fifth above (2), *i.e.* a Twelfth above (1).

(4)   A tone a Fourth above (3), *i.e.* two Octaves above (1).

(5)   A tone a Major Third above (4), *i.e.* two Octaves and a Major Third above (1).

(6)   A tone a Minor Third above (5), *i.e.* two Octaves and a Fifth above (1).

These are the most important members of the series. Their vibration-numbers are connected by a simple law, which is easily deduced from the above relations. If the fundamental tone makes 100 vibrations per second, (2) will make twice as many *i.e.* 200; (3), being a Fifth above (2), will have for its vibration-number $\frac{3}{2} \times 200$, or 300. For (4), which is a Fourth above (3), we get similarly $\frac{4}{3} \times 300$, or 400; for (5) $\frac{5}{4} \times 400$, or 500; for (6), $\frac{6}{5} \times 500$, or 600. Thus the numbers come out 100, 200, 300 and so on, or generally, whatever be the vibration-number of (1), those of (2), (3), (4), &c., are respectively *twice, three times, four times, &c.* as great. If C in the bass clef be selected as the fundamental tone, the series complete up to the tenth tone is shown in musical notation as follows:—
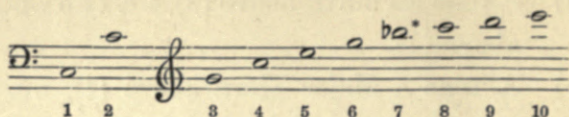
The asterisk denotes that the pitch of the 7th tone is not precisely that of the note by which it is here represented but slightly less acute.
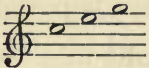
The reader must not suppose that, because the tones into which a note of a musical instrument may usually be decomposed are members of a fixed series, *all* those which we have written down are necessarily present in *every* such note. No more is meant to be asserted than that those which *are* present, be they few or many, must occupy positions determined by the law connecting each tone with its fundamental. The sound may contain, say, (1), (3) and (5) only, or (1), (4) and (8) only, and so on, the rest being entirely absent, but *in no case can a tone intermediate in pitch between any two consecutive members of the series make its appearance.*

44. Experimental evidence shall now be produced in support of the extremely important proposition just enunciated.

We will begin with the sounds of the pianoforte. Let the note be first silently pressed down,

and then ♩ be vigorously struck, and, after three or four seconds, allowed to rise again.   The lower note is at once extinguished, but we now hear its Octave sounding with considerable force from the wires of ♩.   If we permit the damper to fall back on these, by releasing the key hitherto held down, the whole sound is immediately cut off.

Next, retaining the same fundamental note, ♩, let ♩ be quietly freed from its damper, and the experiment repeated as before.   We shall then hear this note sounding on after the extinction of ♩.   Similar results may be obtained with the three next tones, ♩, but they drop off very rapidly in intensity.   The tones above ♩ are so weak as to be practically insensible. The series of tones produced in this succession of trials can only be due to resonance.   But, as has been already shown, the vibrations of any instrument are excited by resonance *only when vibrations of the same period are already present in the surrounding air.*   Accordingly, the only sound *directly* originated in each variation of our experiment, viz. that of the note ♩, must *have contained all the tones suc-*
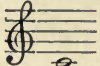
cessively heard. The reader should apply the method of proof here adopted to notes in various regions of the key-board. He will find considerable differences, even between consecutive notes, in the number and relative intensities of the separate tones into which he is thus able to resolve them. The higher the pitch of the fundamental tone, the fewer will the recognisable associated tones become, until, in the region above
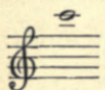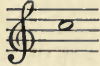
, the notes are themselves approximately

single tones. The causes of these differences will be explained in detail in a subsequent chapter; it is sufficient here to indicate their existence. The result arrived at, thus far, is that the sounds of the piano-forte are in general composite, the number of consti-tuent tones into which they are resolvable being largest in the lower half of the instrument, and dimi-nishing in its upper half, until at last no analysis is called for.

45. The above resolution has been effected by means of the principle of resonance. It can, however, be performed by the ear directly, though only to a small extent and with less ease. In applying the direct analysis to the sounds of a musical instrument, it is best first to produce gently the note correspond-ing in pitch to the tone which it is wished to isolate, and then to develope the compound sound containing
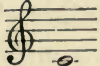
it as a constituent. If the observer has succeeded
in keeping his attention unswervingly fixed on the
note for which he is listening, he will hear it come
out clearly from the mass of tones included in the
composite sound.   If the pianoforte note 𝄢 be
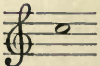
thus examined, the Octave, 𝄞 , and Twelfth,
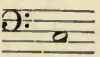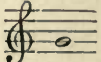
𝄞 , can generally be recognised with consider-

able ease ; the second Octave, 𝄞 , with a little
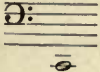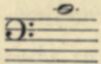
trouble ; the next two tones of the series on p. 86
with increasing difficulty, and those which succeed
them not at all. The reader approaching the pheno-
menon for the first time must not be disappointed if,
on trying this experiment, he fail to hear the tones
he is told to expect.   He should vary its conditions
by changing the note struck, in such a way that his
attention will not be liable to be diverted by the
presence of tones more acute than that of which he

is in search.   Thus a note near 𝄞 may be advan-

tageously chosen to observe the first Octave, 𝄞 ;

one near 𝄢 to observe the Twelfth, 𝄞 ;

one near 𝄢 to observe the second Octave,-

. He may however altogether fail in per-
forming the analysis with the unassisted ear. This
by no means indicates any aural defect, as he may
at first be inclined to imagine. It rather shows
that the life-long habit of regarding the notes of in-
dividual sound-producing instruments as single tones
cannot be unlearned all at once. The case is analo-
gous to that of single vision with two eyes, where *two*
distinct and different images are so blended together
as to appear to all ordinary observation as *one*. The
acoustical observer who is thus situated must rely on
the analysis by resonance, and on the evidence of those
who are able to perform the direct analysis. As he
pursues the subject further experimentally, his ana-
lytical faculty will develope itself.

46.   The composite character of musical sounds,
which we have recognised in the case of the piano-
forte, and shall have ample opportunity of verifying
more generally in the sequel, requires the introduc-
tion here of certain verbal definitions and limitations.
The phraseology hitherto employed, both in the
science of Acoustics and in the theory of Music,
goes on the supposition that the sounds of individual
instruments are single tones, and therefore, of course,
contains no term specially denoting compound sounds
and their constituents. 'Sound,' 'note,' and 'tone'

are used as nearly synonymous.   It will be conve-
nient to restrict the meaning of the latter so that it
shall denote a sound which does not admit of resolu-
tion into simpler elements.   A single sound of deter-
minate pitch will accordingly, in what follows, be
called a *tone*, or *simple tone*.   For a compound sound
the word *clang* will be a serviceable term.   The series
of elementary sounds into which a clang can be re-
solved we shall call its *partial-tones*, sometimes dis-
tinguishing among these, the lowest, or *fundamental
tone*, from the others, or *overtones* of the clang.   This
nomenclature is a direct adaptation of the German
terms employed by Helmholtz.   Its introduction is
due to Professor Tyndall.

   47.   This long discussion has paved the way for
the complete explanation of musical *quality* which is
contained in the following proposition.   *The quality
of a clang depends on the number, orders and rela-
tive intensities of the partial-tones into which it can
be resolved*.   We have here three different causes to
which variations in the quality of composite sounds
are assigned.

   1.   A clang may contain only two or three, or it
may contain half-a-dozen, or even as many as fifteen
or twenty, well developed partial-tones.

   2.   The *number* of partial-tones present remain-
ing the same, the quality of the resulting sound will

vary according as they occupy different positions in the partial-tone series, *i.e.* on their *orders*. Thus, a clang containing *three* tones may consist of (1), (2), (3), or of (1), (3), (5), or of (1), (7), (10), and so on, the *quality* varying in each instance.

3.   The *number* and *orders* of the partial-tones present remaining the same, the quality will vary according to the relative degrees of loudness with which those tones speak. Thus, in the simplest case of a clang consisting of only (1) and (2), either tone may alter in intensity while the other remains constant, and so cause variation in the quality of the sound resulting from their combination.

It is clear that these three classes of variations are entirely independent of each other, that is to say, any two clangs may differ in the number, orders *and* relative intensities, of their constituent partial-tones. The variety of quality thus provided for is almost indefinitely great.   In order to form some idea of its extent, let us see how many clangs of different quality, but of the same pitch, can be formed with the first *six* partial-tones, by variations of *number and order only*.   We will indicate each group of tones by the corresponding figures inclosed in a bracket; thus *e.g.* (1, 3, 5) represents a clang consisting of the first, third and fifth tones.

All the possible groups, each of course contain-

ing the same fundamental tone, are given in the following enumeration :—

Two at a time :

(1, 2), (1, 3), (1, 4), (1, 5), (1, 6).

Total 5.

Three at a time :

(1, 2, 3), (1, 2, 4), (1, 2, 5), (1, 2, 6), (1, 3, 4),
(1, 3, 5), (1, 3, 6), (1, 4, 5), (1, 4, 6), (1, 5, 6).

Total 10.

Four at a time :

(1, 2, 3, 4), (1, 2, 3, 5), (1, 2, 3, 6),
(1, 2, 4, 5), (1, 2, 4, 6), (1, 2, 5, 6),
(1, 3, 4, 5), (1, 3, 4, 6), (1, 3, 5, 6), (1, 4, 5, 6).

Total 10.

Five at a time :

(1, 2, 3, 4, 5), (1, 2, 3, 4, 6), (1, 2, 3, 5, 6),
(1, 2, 4, 5, 6), (1, 3, 4, 5, 6).

Total 5.

Six at a time :   (1, 2, 3, 4, 5, 6).

Total 1.

The whole number of groups is 31, or if we allow the fundamental-tone (1) to count by itself as a sound of separate quality, 32. Let us next examine how many clangs of different quality can be obtained from a single combination of three fixed partial-tones by variations of *intensity only*, supposing that each

tone is capable of but *two* degrees of loudness.   Representing one of these by f, and the other by p, we indicate, *e.g.*, by (f, p, p) a clang in which the fundamental tone is sounded *forte*, and the two overtones *piano*.   The different cases which present themselves are the following :

$$(f, f, f), \; (f, p, f), \; (p, f, f), \; (p, p, f), \; (f, f, p),$$
$$(f, p, p), \; (p, f, p), \; (p, p, p)$$

or *seven* in all, since (p, p, p) has the same quality as (f, f, f).   The number of cases increases very rapidly as we take more partial-tones together. Thus a clang of four tones will produce 15 sounds of different quality ; one of five tones 31 ; one of six tones 63, by variations of intensity only.   Altogether we could form, with six partial-tones, each susceptible of only two different degrees of intensity, upwards of *four hundred* clangs of distinct quality, all having the same fundamental tone.   The supposition above made utterly understates, however, the possible variety of quality dependent only on changes of relative intensity.   A very slight increase or diminution of loudness, on the part of a single constituent tone, is enough to produce a sensible change of quality in the clang.   We should be still far below the mark if we allowed each partial-tone *four* different degrees of intensity, though even this supposition would bring us more than *eight thousand* separate

cases.   Since many more variations of intensity are
practically efficacious, and also since the disposable
partial-tones need by no means be limited to the
first *six*, the above calculation will probably suffice
to convince the reader that the varieties of quality
which the theory we are engaged upon is capable
of accounting for are almost indefinitely numerous.
This is, in fact, no more than we have a right to
demand of the theory, when we reflect on the fine
shades of quality which the ear is able to distinguish.
No two instruments of the same class are exactly
alike in this respect.   For instance, grand pianofortes
by Broadwood and by Erard exhibit unmistakable
differences, which we describe as ' Broadwood tone'
and 'Erard tone.'   Less marked, but still perfectly
recognisable, differences exist between individual in-
struments of the same class and maker, and even
between consecutive notes of the same instrument.
To these we have to add the variations in quality
due to the manner in which the performer handles
his instrument.   Even on the pianoforte the kinds
of tone elicited by a dull slamming touch, and by a
lively elastic one, are clearly distinguishable.   With
other instruments such distinctions are much more
marked.   On the violin we perceive endless grada-
tions of quality, from the rasping scrape of the
beginner up to the smooth and superb tone of a

Joachim. A precisely similar remark applies to wind instruments; the differences, for example, between first-rate and inferior playing on the hautbois, bassoon, horn, or trumpet, being perfectly obvious to every musical ear.

In the next chapter we will discuss the quality and essential mechanism of the principal musical instruments, among which the pianoforte will receive an amount of attention proportionate to its popularity and general use. We begin with the elementary tones of which all composite sounds are made up.

CHAPTER V.

ON THE ESSENTIAL MECHANISM OF THE PRINCIPAL MUSICAL INSTRUMENTS, CONSIDERED IN REFERENCE TO QUALITY.

1. *Sounds of tuning-forks.*

48. When a vibrating tuning-fork is held to the ear, we at once perceive, beside the proper note of the fork, a shrill, ringing and usually rather discordant, sound. If however the fork is mounted on its resonance-box, as in Fig. 24, p. 81, the fundamental tone is so much strengthened that the extraneous sound is by comparison evanescent. Provided the fork be but moderately excited, *e.g.* by gentle stroking with a resined bow, the sound heard is practically[1] a simple tone. It is characterised by extreme mildness, without a trace of anything which could be called harsh or piercing. As compared with

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[1] Except with very gentle bowing, however, or unless the fork's note has been allowed to grow faint, examination with resonators shows *traces* of overtones belonging to the ordinary series given in § 43.

a pianoforte note of the same pitch the fork-tone is wanting in richness and vivacity, and produces an impression of greater depth, so that one is at first inclined to think the fork employed must be an Octave too low.

It is a direct inference from the general theory of the nature of quality that simple tones can differ only in pitch and intensity. Accordingly we find that tuning-forks of the same pitch, mounted on resonance-boxes and set gently vibrating by a resined fiddle-bow, exhibit, whatever be their forms and sizes, differences of *loudness only.* When made to sound with equal intensity by suitable bowing, their tones are absolutely undistinguishable from each other.

2. *Sounds of vibrating strings.*

49. Sounding strings vibrate so rapidly that their movements cannot be followed directly by the eye. It will be well, therefore, that we should examine how the slower and more easily controllable vibrations of non-sounding strings are performed, before treating the proper subject of this section. Take a flexible caoutchouc tube ten or fifteen feet long and fasten its ends to two fixed objects separated from each other by that distance. The tube can be set in periodic vibration by impressing a swaying movement upon it with the hand near either ex-

tremity, in suitable time. According to the rapidity
of the motion thus communicated, the tube will take
up different forms of vibration. The simplest of these
is shown in Fig. 26. *A* and *B* being its fixed ex-
tremities, the tube vibrates *as a whole* between the
two extreme positions *AaB* and *AbB*.

Fig. 26
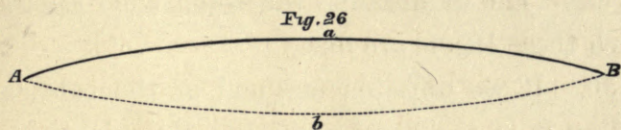
The tube may also vibrate in the form shown in
Fig. 27, where *AabB* and *AcdB* are its extreme
positions.

Fig. 27
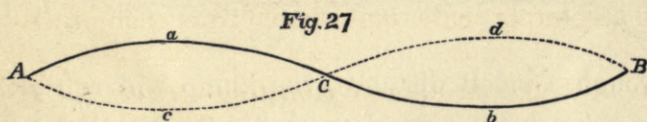
In this instance the middle point of the tube,
C, remains at rest, the loops on either side of
it moving independently, as though the tube were
fastened at *C* as well as at *A* and *B*. For this
reason the point *C* is called a *node*, from the Latin
nodus, a knot. The loops *AC* and *CB* are termed
ventral segments.

Fig. 28 shows a form of vibration with two nodes,

Fig. 28.
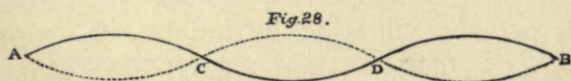
at C and D, dividing the distance AB into three equal ventral segments. We may also obtain forms with three, four, five, &c., nodes, dividing the tube into four, five, six, &c., equal ventral segments, respectively. The stiffness of very short portions of the tube alone imposes a limit on the subdividing process. Let us examine the mechanical causes to which these effects are due.

50. If we unfasten one end of the tube, and, holding it in the hand as in Fig. 29, raise a hump upon it, by suddenly jerking the hand transversely

Fig. 29 .
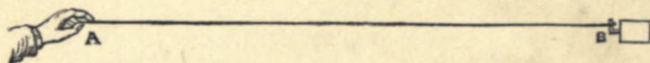
through a small distance, the hump will run along the tube until it reaches its fixed extremity B; it will then be reflected and run back to A, where it will undergo a second reflexion, and so on. *At each reflexion the hump will have its convexity reversed.* Thus, if while travelling from A towards B its form was that of a, Fig. 30, on its way back it will have the form b. After reflexion at A, it will resume its

Fig. 30.
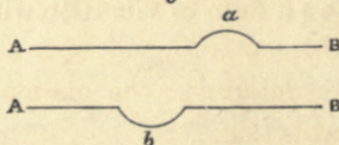
first form a, and so on.   Now, instead of a single
jerk, let the hand holding the free end execute a
series of equal continuous transverse vibrations.   Each
complete vibration will cause a wave, ab Fig. 31, to

Fig. 31.
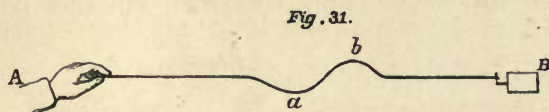
pass along the tube from A to B, where reflexion will
reverse the protuberances, so that the wave will re-
turn from B to A stern foremost.   Next let the tube
be again fastened at both ends, as before, and the
vibrations of the hand impressed at some intermedi-
ate point, as C, Fig. 32.

Fig. 32.
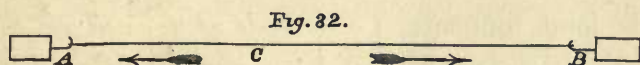
Two sets of waves will now start from C in the
directions of the arrows.   They will be reflected at A
and B, and then their effects will intermingle.   Let us
suppose that the tube has been set in steady motion
and, on the removal of the hand, continues its vibra-
tions without any external force acting on it.   Two
sets of equal waves are now moving with equal ve-
locities from A towards B and from B towards A,
and we have to determine the motion of the tube
under their joint action.

Suppose that a crest[1] *a*, Fig. 33, moving from *A* towards *B*, meets an equal trough[1] *b*, moving from *B* towards *A*, at the point *c*. A particle of the tube situated at this point is solicited by equal forces in opposite directions, and therefore remains at rest.

Fig.33
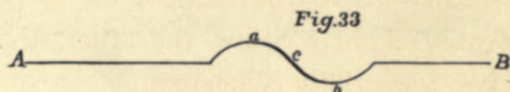
The two opposite pulses then proceed to cross each other, but, as *a* moves to the right and *b* to the left with equal speed, there is nothing to give either of them at the point *c* an influence superior to that exerted in the contrary direction by the other. The particle at *c* therefore *remains at rest* under their joint influence, *i.e. a node is formed at that point*. If a trough had been moving from *A* towards *B*, and an equal crest from *B* towards *A*, the result would clearly have been the same.

A node must therefore be formed at every point where two equal and opposite pulses, a crest and a trough, meet each other.

51. The annexed figure represents two series of equal waves advancing in opposite directions with

[1] Provided that confusion with water-waves be explicitly guarded against, there is no objection to retaining this convenient phraseology for distinguishing between opposite protuberances.

equal velocities. The moment chosen is that at which crest coincides with crest and trough with trough. The joint effect thus produced does not appear in the figure, our object at present being merely to determine the number and positions of the resulting nodes. For the sake of clearness, one set of waves is represented slightly below the other, though in fact the two are strictly coincident.

Fig. 84.
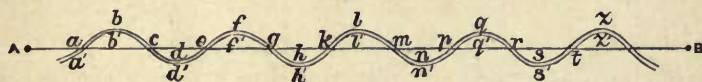
Let the waves $abdf...z$ be moving from left to right, the waves $z'ts'q'...a'$ from right to left. The crest klm meets the trough $pn'm$ at m. After these have crossed each other, the trough ghk and the crest $rq'p$ will also meet at m, since km and pm are equal distances. Similarly the crest efg and the trough $ts'r$ will meet at m. Accordingly the point m is a node, and, by exactly the same reasoning, so are a, c, e, g, k, p, r, t, &c. The distances between pairs of consecutive nodes are all equal, each being a single pulse-length, *i.e.* half a wave-length, of either series.

Two pulse-lengths, as gk and km, give *three* nodes g, k, and m; three pulse-lengths four nodes, and so on. There is thus always one node more than the number of pulses. On the other hand, the fixed ends

of the tube, which are the origins of the systems of
reflected waves, occupy two of these nodes. Deducting
them we arrive at this result :—

*The number of nodes is one less than the number
of the pulse-lengths (or half wave-lengths), which to-
gether make up the length of the vibrating tube.*

52.   We will now ascertain how the portions of
the tube between consecutive nodes move while the

Fig. 35.
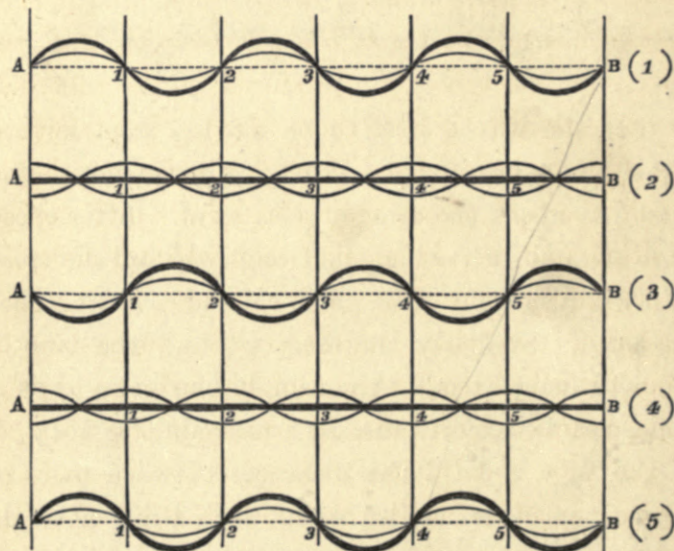
two systems of waves are simultaneously passing
along it. Let *A* and *B*, Fig. 35, be the fixed ends, as
before, and let us take five nodes at the points 1, 2,
3, 4, 5. In (1), the systems of waves coincide, ac-

cordingly each point of the tube is displaced through twice as great a distance as if it had been acted on by only one system. The tube thus takes the form indicated by the strong line in the figure. In (2), one set of waves has moved half a pulse-length to the right, and the other the same distance to the left. The two systems are now in complete antagonism, the displacements being equal in amount and opposite in direction at every point. The tube is therefore momentarily in its undisturbed position. In (3), each system has moved through a pulse-length, and the maximum effect is again produced on the tube, but in opposite directions to those of (1). In (4), where the systems have moved through a pulse-length and a half, the tube passes again through its undisturbed position, and, in (5), regains that which it occupied in (1), the systems of waves, meanwhile, having each traversed two pulse-lengths, or one wave-length. Thus the tube executes one complete vibration in the time occupied by a pulse in passing along a length of the tube equal to *twice one of its own ventral segments*. In other words, *the tube's rate of vibration varies as the number of segments into which it is divided*. It moves most slowly in the form shown in Fig. 26 with but a single segment; twice as fast in that of Fig. 27, where it is divided into two segments; three times as fast with three segments, and so on.

It is easy to confirm this by direct experiment, the
swaying movement of the hand on the tube needing
to be twice as rapid for a form of vibration with two
segments as for a form with one, and so on.

53.   Instead of comparing the different rates at
which the same tube vibrates when divided into
different numbers of ventral segments, we may com-
pare the rates of vibration of tubes of different
lengths divided into the same number of segments.

Let us take as an example the two tubes AB,
CD, Fig. 36, each divided by three nodes into four

Fig. 36
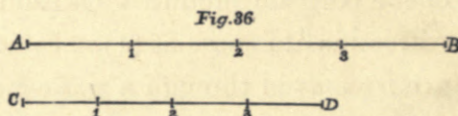
ventral segments.   By what has been already shown,
the time of vibration of either tube will be that
which a pulse occupies in traversing two of its ven-
tral segments.   Therefore the time of vibration of
AB will be to that of CD as $A2$ is to $C2$, *i.e.* as one
half of AB is to one half of CD, or as AB is to CD.
This reasoning is equally applicable to any other
case.   Accordingly we have the general result that,
when tubes of different lengths are divided into the
same number of ventral segments, their times of
vibration are proportional to the lengths of the
tubes, or, which comes to the same thing, their *rates*

of vibration are inversely proportional to their lengths.
The reader should observe that it has been through-
out this discussion assumed that the *material, thick-
ness* and *tension* of the tube, or tubes, in question
were subject to no variation whatever. Any changes
in these would correspondingly affect the rates of
vibration produced.

54. We are now prepared to examine the motion
of a sounding string. Its ends are fastened to fixed
points of attachment and the string is excited at
some intermediate point, by plucking it with the
finger, as in the harp and guitar, by striking it with
a soft hammer, as in the pianoforte, or by stroking it
with a resined bow, as in the violin and other instru-
ments of the same class. The impulses thus set up
are reflected at the extremities of the string (in the
violin at the bridge and at the finger of the per-
former) and behave towards each other exactly as
in the case of the vibrating tube considered above.
The results there at once obtained are accordingly
applicable to the case before us. The string may
vibrate in a single segment as in Fig. 26. This is
the form of slowest vibration with a string of given
length, material and tension. Accordingly, when
thus vibrating, the string produces the deepest note
of which, all other conditions remaining the same,
it is capable. The string may also vibrate in the

forms shown in Figs. 27, 28, 35, 36, or in forms
with larger numbers of segments. The rapidity of
vibration in any one of these forms is, as we have
seen [§ 52], proportional to the number of segments
formed, so that, with two segments, it vibrates
twice, with three, *thrice*, with four, *four times*, as fast
as in the form with one segment. It follows hence
[§ 43] that the notes obtained by causing a string to
vibrate successively in forms with 1, 2, 3, 4, 5, &c.,
segments are all partial-tones of one compound sound,
the lowest being of course its fundamental-tone.

The modes of eliciting the sounds of stringed
instruments described on the preceding page are
not capable of setting up any *one* of the above forms
of vibration *by itself*, but give rise to a movement
which is the resultant of several such vibration-
forms compounded together. Each separate vibra-
tion-form thus called into existence sings, as it were,
its own note, without heeding what is being done
by its fellows. Accordingly, a certain number of
tones belonging to one family of partial-tones are
simultaneously heard.

How many, and which, members of the general
series of partial-tones are present, and with what
relative intensities, in the sound of a string set
vibrating by a blow, depends on the position of
the point at which the blow is delivered, on the

nature of the striking object, and on the material of the string. It is clear that a node can never be formed at the point of percussion. Therefore no partial-tone requiring for its production a node in that place can exist in the resulting sound. If, for instance, we excite the string exactly at its middle point, the forms of vibration with an *even number* of ventral segments, all of which have a node at the centre of the string, are excluded, and only the *odd* partial-tones, *i.e.* the 1st, 3rd, 5th, and so on, are heard. In this manner we can always prevent the formation of any assigned partial-tone, by choosing a suitable point of percussion. On the other hand, a vibration-form is in the most favourable position for development when the *middle point* of one of its ventral segments coincides with the point of percussion. The more nearly it occupies this position the louder will be the corresponding partial-tone, while the further it recedes from this position towards that in which one of its nodes falls on the point of percussion, the weaker will the partial-tone become.

The form and material of the hammer, or other object with which the string is struck, have also a great influence in modifying the quality of the sound produced. Sharpness of edge and hardness of substance tend to develope high and powerful overtones, a rounded form and soft elastic substance

to strengthen the fundamental-tone. The material of the string itself produces its effect chiefly by limiting the number of partial-tones. Its stiffness resists division into very short segments, and this implies, for every string, a fixed limit beyond which further subdivision ceases and where, therefore, the series of overtones is cut off. Hence very thin mobile strings are favourable, thick weighty strings unfavourable, to the production of a large number of partial-tones.

55. Having examined what determines the *quality* of the sound of a vibrating string, we have next to enquire on what its *pitch* depends. This term is indeed, strictly speaking, inappropriate to a composite sound containing a series of different tones each having its own vibration-number and independent position in the musical scale. If, however, we use the phrase 'pitch of a sound' as equivalent to 'pitch of the fundamental-tone of the sound,' we shall avoid any confusion arising from this circumstance. The pitch of a string-sound depends of course on the rate at which the string is vibrating. We have seen that when the material, thickness and tension of a string remain the same, its rate of vibration varies inversely as the length of the string. Accordingly, *the vibration-number of a string-sound varies inversely as the length of the string.* It follows hence that the

relations which connect the vibration-numbers of sounds forming given intervals with each other, hold equally for the lengths of the strings by which those sounds are produced.   To verify this by experiment we have only to stretch a wire between two fixed points A and B, Fig. 36 *bis*, and divide it into two segments by applying a bridge to it at some inter-mediate point C.   If AC bears to CB any one of the simple numerical ratios exhibited in the table on

Fig. 36 (*bis*.)
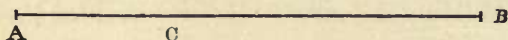
p. 69, we obtain the corresponding interval there given by alternately exciting the vibrations of the two segments at any pair of points in AC and CB respectively.   Thus, if CB is twice as long as AC, the sound produced by the former will be one Octave lower than that produced by the latter.   If AC is to CB in the proportion of 2 to 3, AC's sound will be a Fifth above CB's ; and similarly in other cases.   It was by experiments of this kind that the ancient Greek philosopher, Pythagoras, is said to have made out the existence of a connexion between certain musical intervals and the ratios of certain small integers.   Thus an Octave was produced by a wire divided into two parts in the proportion of 2 to 1 ; a Fifth was obtained by division in the proportion

of 3 to 2, and so forth. The relations existing between these lengths and the vibration-numbers of the notes produced by them were, however, entirely unknown to Pythagoras and his contemporaries; indeed it was not until the seventeenth century that they were discovered by Galileo.
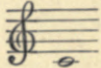
In instruments of the violin class, the pitch of the notes varies according to the position of the finger on the vibrating string. The length of string intercepted between the fixed bridge and the finger admits of being altered at pleasure, and thus every shade of pitch can be produced from such instruments. The resined bow maintains the vibration of the string by alternately dragging it out of its position of rest, letting it fly back again, catching it once more, and so on. The hollow cavity of the instrument reinforces the string-sound by resonance. The quality of instruments of the violin class is vivacious and piercing. The first eight partial-tones are well represented in their sounds.

The Pianoforte.

56. In this instrument each wire is stretched between two pegs, which are fixed into a flat plate of wood called the sound-board. The string is fastened to one peg, and coiled round the other which admits of being turned about its own axis by means

of a key of suitable construction. This allows the
string to be accurately tuned, since by tightening
or loosening the wire we raise or lower its pitch
at pleasure. In small instruments *two*, in larger
ones *three*, wires in unison with each other usually
correspond to each note of the key-board. While
the pianoforte is not in action a series of small
pieces of wood covered with list, called 'dampers,'
rest upon the wires. These are connected with the
key-board in such a manner that, when a note is
pressed down, the corresponding damper rises from
its place, and the wires it previously covered remain
free until the note is allowed to spring up again,
when the damper immediately sinks back into its
original position. Each note is connected with an
elastic hammer, which deals a blow to its own set of
wires and then springs back from them. The wires
thus set in motion continue to vibrate until either
the sound gradually dies away, or is abruptly extin-
guished by the descent of the damper. The action of
the two pedals is as follows : the soft pedal shifts the
key-board and associated hammers in such a way that
each hammer only acts on *one*, or on *two*, of the wires
corresponding to it, instead of on its complete set of
two or three wires. The sound produced by striking
a note is therefore proportionally weakened. The
loud pedal lifts all the dampers off the wires at once.
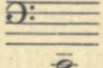
It thus not only allows notes to continue sounding after the fingers of the player have quitted them, but places other wires than those actually struck in a position to sound by resonance. The number of wires thus brought into play by striking a single note of the instrument will be easily seen to be considerable. Suppose, first, that a *simple tone, e.g.* that of a tuning-fork, is sounding near the wires of a pianoforte with the loud pedal *down*, its pitch being that of middle *C*, ; the wires of the corresponding note will of course resonate with it, vibrating in the simplest form with only one ventral segment. The wires of the note , one Octave below it, are also capable of producing middle *C* when they vibrate in the form with two segments. So are those of , a Twelfth below it, when vibrating with three segments; those of , two Octaves below it, vibrating with four segments, and so on. Proceeding in this way, we determine a series of notes on the key-board of the pianoforte the wires of which are able to produce a simple tone of the pitch of middle *C*. They obviously follow the same law as the harmonic overtones of a compound sound with middle *C* for its fundamental-tone, except that

the successive intervals are reckoned *downwards* instead of upwards. The wires of all these notes will reinforce the tone of the tuning-fork by resonance. If now we remove the fork and strike middle *C* on the pianoforte itself, we obtain of course a sound consisting of a number of simple tones. To *each* of these latter there corresponds a descending series of notes on the key-board determined in the manner just explained. A full chord struck in the middle region of the instrument will, therefore, command the more or less active services of many more wires than have been set vibrating by direct percussion. The increase of *loudness* thus secured is not very considerable, the effect being rather a heightened richness, like that of a mass of voices singing pianissimo. The sustaining power of the loud pedal renders care in its employment essential. It should, as a general rule, be held down only so long as notes belonging to one and the same chord are struck. Whenever a change of harmony occurs, the pedal should be allowed to rise, in order that the descent of the dampers may at once extinguish the preceding chord. If this precaution is neglected, perfectly irreconcilable chords become promiscuously jumbled together, and a series of jarring discords ensue which are nearly as distressing to the ear as the actual striking of wrong notes. The quality of

pianoforte notes varies greatly in different parts of the scale. In the lower and middle region it is full and rich, the first six partial-tones being audibly present, though, 4, 5, 6 are much weaker than 1, 2, 3. Towards the upper part of the instrument the higher partial-tones disappear, until in the uppermost Octave the notes are approximately simple-tones, which accounts for their tame and uninteresting character. The pianoforte shares with all instruments of fixed sounds certain serious defects which will be discussed in detail in a subsequent chapter.

When a vibrating wire is passing through its undisturbed position, its tension is necessarily some-what less than at any other moment, since, in order to assume the curved segmental form, it must be a little elongated, which involves a corresponding in-crease of tension. Hence the two pegs by which the ends of a wire are attached to the sound-board are submitted to an additional strain *twice* during each complete segmental vibration. The sound-board, being purposely constructed of the most elastic wood, yields to the rhythmic impulses acting upon it, and is thrown into segmental vibrations like those of the wire.

These vibrations are communicated to the air in contact with the sound-board, and then transmitted further in the ordinary way. The amount of surface

which a wire presents to the air is so small that, but
for the aid of the sound-board, its vibrations would
hardly excite an audible sound.   The reader will not
fail to notice that the sound-board of the pianoforte
plays the same part as the hollow cavity of the violin,
and is in fact a *solid resonator*.   In the harp, the
framework of the instrument serves the same purpose.
We have, in this combination of a vibration-exciting
apparatus with a resonator, the type of construction
adopted in nearly all musical instruments.

3.   *Sounds of organ-pipes.*

57.   It has been shown [§ 51] that, when two
series of equal waves due to transverse vibrations
travel along a stretched wire in opposite directions,
stationary nodes are formed at equal distances along
it, separated by vibrating segments of equal lengths.
Let us now suppose that two series of equal waves
due to *longitudinal* vibrations are traversing, in oppo-
site directions, a column of air contained in a tube of
uniform bore.   Each set of such waves has its own
associated wave-form [§ 18, p. 37].   We have only,
therefore, to consider the curves drawn in Fig. 35 as
associated waves corresponding to longitudinal air-
vibrations, in order to make the conclusions of § 52
at once applicable to the case in hand.   The result
is a series of equidistant nodes, or points of per-

manent rest, distributed along the air-column. The intervening portions of air vibrate longitudinally at the same rate as the corresponding ventral segments of Fig. 35. We have here, as in the case of the sounding wire, all the conditions for the production of a musical note, of pitch corresponding to the rapidity of vibration obtained. It only remains to show that, in the case of every organ-pipe, two sets of equal waves traverse in opposite directions the air-column which it contains.

Organ pipes are of two kinds, called respectively 'stopped' and 'open,'—terms which, however, apply only to one end of the pipe; the other is in both kinds open.

To begin with first variety :
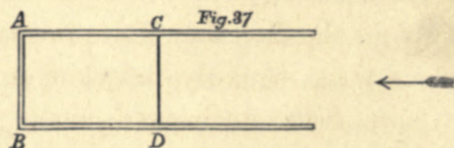
Fig. 37

58. Let *AB*, Fig. 37, be the closed end of a stopped pipe, and let a series of pulses of condensation and rarefaction be passing along the air within it in the direction shown by the arrow. First let a pulse of condensation, *CABD*, have just reached *AB*. By supposition, the air in *CABD* is denser, and therefore at a higher pressure, than that behind it. It

will therefore expand.   Forward motion being barred by *AB*, the expansion must take place entirely in the opposite direction.   Hence the pulse of condensation is reflected at the end of the pipe, and proceeds to describe its previous course in the reverse direction.   Next, suppose *CABD* to be a pulse of rarefaction.   The air in it is at a less pressure than that of the air behind it.   As there is a fixed obstacle in front, this rarefaction must be wholly filled up from behind.   The condensed air behind will expand more during the process, and become itself more rarefied, than if there had been no obstacle.   A rarefied pulse will therefore return along the pipe.   Thus a pulse of rarefaction, equally with one of condensation, is reflected at the closed end of the pipe.   Neither pulse suffers any other change except in the direction of its motion.   Since every pulse is thus regularly reflected at *AB*, and made to travel back unchanged along the pipe, it follows that a system of equal waves advancing in the direction of the arrow is necessarily met by an exactly equal system proceeding in the opposite direction.   For stopped pipes, therefore, the point required to be proved is made out.

Let *AB*, Fig. 38, be one end of an open pipe, along which condensed and rarefied pulses are being alternately transmitted in the direction of the arrow.

First, let *CABD* be a pulse of condensation which has just reached *AB*. If the pipe had been continuous, the particles of air near *AB* would have
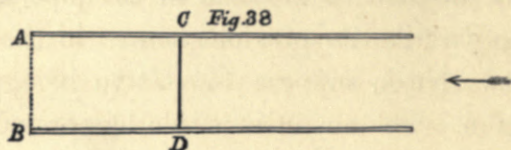
moved in the direction of the arrow till they were stopped by the increasing pressure in their front. This would correspond to the instant of greatest density at *AB*. They would then begin to move in the direction opposite to that of the arrow, the pressure in front *i.e.*, on the left in the figure, being greater than that behind. As, however, the pipe is open and the air free to move in all directions, the condensation just outside its mouth is never great : the particles will move further in the direction of the arrow before being stopped than they would do in the case of a continuous tube and will, therefore, leave behind them a region in which the air is more rarefied than it would have been at the same time in the corresponding part of the continuous tube. This region of rarefied air will produce a pulse of rarefaction which will be transmitted in the direction opposite to that of the arrow. Conversely, when a pulse of rarefaction reaches the open end of the

pipe, the rarefaction just within its mouth cannot become great, since the air is free to enter from all sides.   The particles behind, which in the case of a continuous tube would move in the direction of the arrow and diminish the rarefaction, will therefore be sooner stopped : the pressure and density will be greater than at the corresponding place and time in the continuous tube : and thus a pulse of condensation will be produced and transmitted back along the pipe.

To look at the matter from a somewhat different point of view, the motion of the air in the open pipe will be practically the same as it would have been if the pipe had formed part of a continuous tube, but air had been removed at the point corresponding to the opening whenever a pulse of condensation arrived there and air had been injected at the same point whenever a pulse of rarefaction reached it.

By the above reasoning[1] which the student should carefully compare with that of § 21, it is clear that reflexion takes place at an open as well as at a closed end of a pipe : with this important difference, how-ever, that in the case of the open orifice condensation is turned into rarefaction and rarefaction into con-

[1] I am indebted for this popular explanation of reflexion at the mouth of an open pipe to Mr Coutts Trotter, Senior Fellow of Trinity College, Cambridge.

densation, so that the wave returns hind part foremost.

59. We will now examine what forms of segmental vibration the air in a stopped pipe can adopt. Every such form must necessarily have a *node coincident with the closed end of the pipe*, since no longitudinal vibrations are possible there. The impulses constituting the series of direct waves are not, as we shall see presently, originated, like those of a pianoforte string, at some intermediate point, but enter the pipe *at its open end*. This must therefore be a point of maximum vibration. Now a glance at Fig. 35, shows that the maxima of vibration are at the middle points of the ventral segments. Hence *the centre of a segment must coincide with the open end of the pipe*.

The above considerations suffice to solve the problem before us. If the closed end of the pipe is placed at *A*, Fig. 35 (1), the open end must be midway between *A* and 1, or between 1 and 2, 2 and 3, 3 and 4, and so on. No other forms of vibration are possible.

Fig. 39 shows the air in a stopped pipe of given length vibrating in four such ways. The vertical lines indicate the positions of the nodes. For the sake of greater clearness, the loops of the associated vibration-forms are in each case drawn in dotted lines.

Fig. 89.

(A)

(B)

(C)

(D)
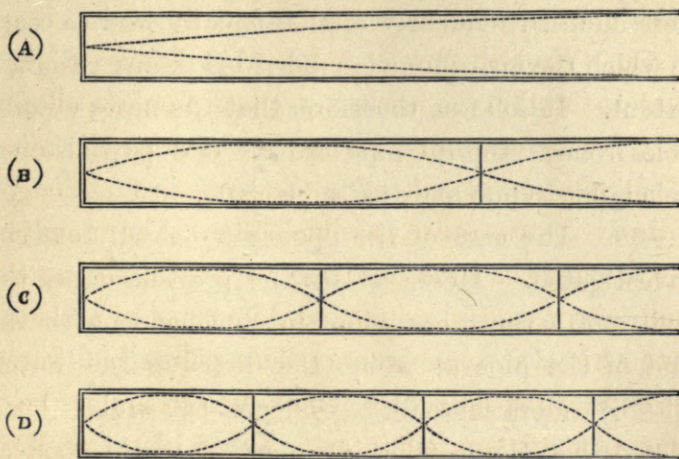
In (A) we have half a segment; in (B) a segment and a half; in (C) two segments and a half; in (D) three segments and a half. The numbers of segments into which the air-column is divided in these four cases are, therefore, proportional to

$$\tfrac{1}{2}, \ 1\tfrac{1}{2}, \ 2\tfrac{1}{2}, \ 3\tfrac{1}{2},$$

i.e. to $\tfrac{1}{2}, \ \tfrac{3}{2}, \ \tfrac{5}{2}, \ \tfrac{7}{2},$

or to the whole numbers 1, 3, 5, 7.

Now by § 53 it appears that the rate of vibration in any form varies as the number of segments which it contains. The vibration-numbers of the sounds produced in the present instance are, therefore, proportional to 1, 3, 5, 7, respectively, *i.e.* we get the first four *odd* partial-tones of a sound of which (A)

gives us the fundamental-tone [§ 43]. The reasoning here adopted evidently applies equally well to cases in which the air-column is subdivided to any assigned extent. It follows, therefore, that the notes obtainable from a stopped pipe are all odd partial-tones belonging to one and the same clang.

60. The case of the open pipe shall next be investigated. Here, as in the previous case, the centre of a ventral segment must coincide with the end of the pipe at which the direct pulses enter. The considerations alleged on pp. 120 and 121 indicate that the same thing must also hold good at the opposite orifice[1].

Referring once more to Fig. 35 (1), we obtain all the possible modes of vibration which satisfy both the above conditions by placing one end of the pipe midway between *A* and 1, and the other successively half way between 1 and 2, 2 and 3, 3 and 4, and so on. The first four of the cases thus obtained, for a tube of constant length, are shown in the next figure, which is drawn on precisely the same plan as Fig. 39.

In each case, the two half-segments at the ends

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[1] I here assume, for the sake of simplicity, what is not rigorously true, viz. that reflexion at an open end of a pipe is as complete as at a stopped end. It is however approximately true for the pipes employed for musical purposes, whose transverse dimension is small compared to their length.

of the pipe make up one whole segment.   The num-
bers of segments into which the air-column is divided
are, therefore, in (*A*), 1 ; in (*B*), 2 ; in (*C*), 3 ; in
(*D*), 4.   The same law would obviously hold for
higher subdivisions.   Hence, in the case of an open
pipe, the rates of all the possible modes of segmental

Fig.40.
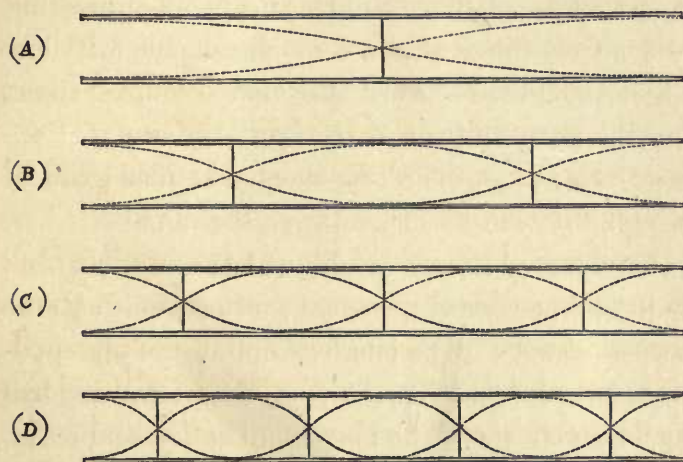
vibration are as the numbers 1, 2, 3, 4, 5, &c.   The
notes obtainable from such a pipe are, therefore,
the *complete* series of partial-tones belonging to one
and the same clang.

61.   If the *slowest* forms of vibration, shown at
(*A*) in Figs. 39 and 40, are compared with each other,
it will be at once seen that the vibrating segment of
Fig. 39 is exactly twice as long as that of Fig. 40.

Hence, *the gravest tone obtainable from a stopped pipe is always one Octave lower than the gravest tone producible from an open pipe of the same length.* It has been shown in § 39 that this result of theory is borne out by experiment.

62.   In order to complete this investigation, it is necessary to determine the pitch of the lowest note which a pipe of given length is capable of uttering. By § 52 we know that a complete segmental vibration is performed during the time occupied by a pulse in traversing *twice the length of a single segment.* In (*A*) Fig. 39, this is equal to *four times the length of the tube.* The velocity of the pulse is here the velocity of Sound in air, which, under ordinary conditions of temperature, &c., we may put at 1125 feet per second[1]. The vibration-number of a stopped pipe's lowest tone is therefore found by dividing 1125 by four times the length of the pipe expressed in feet. Conversely the length of a stopped pipe which is to have as its deepest tone a note of given pitch, is found by dividing 1125 by four times the vibration-number of the note to be produced. The quotient gives the required length in feet. For example, middle *C* of the pianoforte makes 264 vibrations per second. The required length in this

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[1] Tyndall's *Sound*, p. 23.

case would be expressed by $\dfrac{1125}{1056}$, which is rather more than 1 ft. $\frac{1}{2}$ in., *i.e.* roughly speaking, *one foot*. An *open* pipe, to produce the same note, would therefore have to be *two* feet in length.

It has been just shown that the vibration-number of the lowest tone producible, either from a stopped or an open pipe, varies inversely as the length of the pipe. The length of the pipe therefore varies inversely as the vibration-number. Hence the relations established in § 55 for strings hold also for columns of air contained in pipes. The case of the pipe-sounds is, however, somewhat simpler than that of the string-sounds, since the pitch of the latter depends on the tension of the strings as well as on their lengths, whereas, in the former, pitch depends, under given atmospheric conditions, on length alone. Hence we may define a note of assigned pitch by merely stating the length of the stopped, or open, pipe whose fundamental tone it is. The open pipe is commonly preferred for this purpose, and accordingly organ builders call middle C '2-foot tone;' the Octave below it '4-foot tone,' and so on. The lowest C on modern pianofortes is '16-foot tone;' that one Octave lower, which is found only on the very largest organs, '32-foot tone.' The highest note of the pianoforte, usually A, would be about '2-inch tone.'

63. The reader should observe that, in the course of this discussion, we have incidentally obtained a more complete theory of resonance than could be given in chapter III. When a tuning-fork is held at the orifice of a tube, the strongest resonance will be produced if the note of the fork coincides with the fundamental tone of the tube. A decided, though less powerful, resonance ought also to ensue if the fork-note coincides with one of the higher tones of the tube, which, as we know, are all overtones of its fundamental. A resonance-box is only a stopped[1] pipe under another name. We may therefore employ it to test the truth of our result, that the only tones obtainable from a stopped pipe are the *odd* partial-tones of a clang of which the first is the fundamental-tone. I possess a series of forks giving the first seven partial-tones of a clang. When I strike 1, 3, 5 or 7, and hold them before the open end of the stopped resonance-box corresponding to 1, a decided rein-forcement of their tones is heard. If I do the same with 2, 4, or 6, hardly any resonance is produced. Thus our theoretical result is experimentally verified.

64. Organ pipes are divided into two classes according as their sounds are originated.

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[1] For forks of high pitch it is, however, found [best to use resonance-boxes open at both ends, and therefore corresponding to open pipes.

(1)   by blowing against a sharp edge,

(2)   by blowing against an elastic tongue.

Those of the first class are called *flue*-pipes ; those of the second class *reed*-pipes. We will consider each class by itself.

65.   *Flue-pipes.* Here the wind is driven through a narrow slit against a sharp edge placed exactly opposite to it, in the manner shown in Fig. 41, which represents a vertical section of a portion of the pipe near the end at which its sound originates.

Fig. 41.
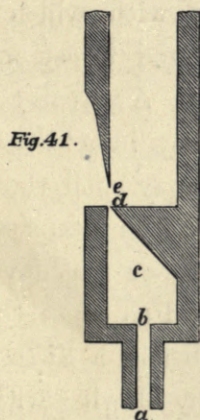
The air is forced by the bellows through the tube *ab* into the chamber *c*, and escapes through the slit *d*, thus impinging against the edge *e*, and exciting in the air-column above segmental vibrations whose period is controlled by the length of the pipe.

The regular musical note thus produced is accompanied by a hissing sound, which may be imitated by blowing with the mouth against a knife-edge held in front of it. This sound however contributes nothing to the general effect, being, where a pipe is properly constructed, inaudible except in its immediate neighbourhood.

Stopped wooden flue-pipes of large aperture, blown by only a light pressure of wind, produce sounds which are nearly simple tones ; only a trace of partial-tone No. 3 being perceptible. Such tones, like the fork-tones with which they are in fact almost identical, sound sweet and mild, but also tame and spiritless. A greater pressure of wind develops 3 distinctly in addition to 1, and, if it becomes excessive, may spoil the quality by giving the overtone too great an intensity compared to that of the fundamental, or may even extinguish the latter altogether, and so cause the whole sound to jump up an Octave and a Fifth. This result may easily be obtained by blowing with the mouth into a small 6-inch stopped pipe, which can readily be procured at any organ factory.

Stopped pipes of narrow aperture develope the third and fifth partial-tones with distinctness.

In the case of an open pipe the fundamental-tone is never produced by itself. According to the dimen-

sions of the pipe, and the pressure of wind, it is accompanied by from two to five overtones. Open flue-pipes present, therefore, various degrees of *timbre* which are exhibited in the different 'stops' of a large organ.

66. *Reed-pipes.* The apparatus by which the sounds of pipes of this class are originated is the following. One end of a thin narrow strip of elastic metal, called a 'tongue,' is fastened to a brass plate, while the other end is free. A rectangular aperture, very slightly larger than the tongue, is cut through the plate, so as to allow the tongue to swing into and out of the aperture, like a door with double hinges, without touching the edges of the aperture as it passes them. The accompanying figure shows this piece of mechanism, which is called a 'reed,' in its position of rest.

Fig. 42.

It is set in motion by a current of air being driven against the free end of the tongue, which is thus made to swing between limiting positions as shown in the annexed sections.

Fig. 43.

(A)   (B)
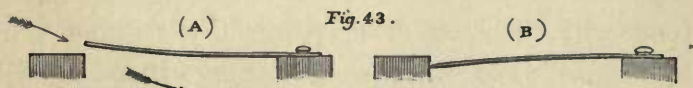
When the tongue occupies any position inter-
mediate between that of (*A*) and its position of equi-
librium, air is blown through the aperture in the
direction indicated by the arrows in (*A*).   From the
moment that the tongue passes through its equi-
librium-position towards that shown in (*B*), the current
of air is barred by the accuracy with which the tongue
fits into the aperture beneath it.   Only when the
tongue again emerges can the air resume its passage.

A series of equal impulses of air are thus produced
at equal intervals of time and the instrument, there-
fore, gives rise to a regular musical sound.   Its clang
is highly composite containing distinctly recognisable
partial-tones up to the 16th or 20th of the series.
Thus a reed does not require to be associated
with a resonating column in order to produce a
musical sound ; in fact the instrument called the
harmonium consists of reeds without such adjuncts.
The *timbre* of an independent reed is, however,
characterised by too great intensity on the part
of the higher partial-tones.   It is desirable to correct
this defect by strengthening the fundamental-tone of
the clang.   This is done by placing the reed in the
mouth of an open pipe whose deepest tone coincides
with the fundamental-tone of the reed-clang.   This
tone will then be most powerfully reinforced by
resonance.   The other partial-tones of the clang will

also be strengthened by resonance, but to a smaller and smaller extent as their *order* rises. The force required to throw a column of air into rapid vibration is greater than that which suffices to set up a slow vibration. Since the force necessary to produce segmental vibration increases very rapidly as the subdivisions of the air-column become more numerous, the very high partial-tones of the reed-clang are practically unsupported by the resonance of the associated pipe. We shall see in the sequel how it happens that the quality of the resulting sound is improved by this circumstance.

It is clear that sounds varying widely in quality may be obtained by associating a reed with pipes of varying lengths and forms. The resulting quality will be quite different according as the lowest tone which the pipe is capable of strengthening by resonance coincides with the fundamental-tone, or with one of the overtones, of the reed-sound. The *form* of the pipe may also be modified, so as to be conical or of any other shape, which will bring in other changes in its resonating properties. In these ways we have provision for the great variety of quality among reed-pipes which we find represented in organ stops of that class.

4. *Sounds of orchestral wind-instruments and of
the human voice.*

67. The *flute* is in principle identical with an
open flue-pipe. The lips, and a hole near the end
of the tube, play the parts of the narrow slit and
opposing edge. The quality of the instrument is
sweet, but too nearly simple to be heard during a
long solo without becoming wearisome. Its best
effects are produced by contrast with the more
brilliant *timbre* of its orchestral colleagues.

The *clarionet, hautbois* and *bassoon* have wooden
reeds. The clarionet has a cylindrical tube, the
hautbois and bassoon have conical tubes.

In the *horn* and *trumpet* the lips of the per-
former supply the place of a reed.

68. The apparatus of the *human voice* is essen-
tially a reed (the vocal chords) associated with a
resonance-cavity (the hollow of the mouth).

The vocal chords are elastic bands situated at
the top of the wind-pipe, and separated by a narrow
slit, which opens and closes again with great exact-
ness as air is forced through it from the lungs. The
form and width of the slit allow of being quickly
and extensively modified by the changing tension of
the vocal chords, and thus sounds widely differing
from each other in pitch may be successively pro-

duced with surprising rapidity. In this respect, the human 'reed' far exceeds any that we can artificially construct.

The size and shape of the cavity of the mouth may be altered by opening or closing the jaws, raising or dropping the tongue and tightening or loosening the lips. We should expect that these movements would not be without effect on the resonance of the contained air, and this proves on experiment to be the fact. If we hold a vibrating tuning-fork close to the lips, and then modify the resonating cavity in the ways above described, we shall find that it resounds most powerfully to the fork selected when the parts of the mouth are in one definite position. If we try a fork of different pitch, the attitude of the mouth, for the strongest resonance, is no longer the same.

Hence, when the vocal chords have originated a reed-sound containing numerous well-developed partial-tones, the mouth-cavity, by successively throwing itself into different postures, can favour by its resonance first one partial-tone, then another; at one moment *this* group of partial-tones, at another *that*. In this manner endless varieties of quality are rendered possible. Good vocalisation, therefore, requires the resonating cavity to be so placed as to modify in the way most attractive to the ear

the quality of the sounds produced by the vocal chords.

The complete analysis of the sounds of the human voice into their separate partial-tones presents peculiar difficulties to the unassisted ear, and can hardly be effected without the help of resonators such as those described in § 42. By their aid we can detect in the lower notes of a bass voice, when vigorously produced, shrill overtones reaching as far as No. 16, which is four Octaves above the fundamental-tone. Under certain conditions these high overtones can be readily heard without recourse to resonators. When a body of voices are singing *fortissimo* without any instrumental accompaniment, a peculiar shrill tremulous sound is heard which is obviously far above the pitch of any note professedly being sung. This sound is, to my ears, so intensely shrill and piercing as to be often quite painful. I have also observed it when listening to the lower notes of a powerful contralto voice. The reason why these acute sounds are tremulous will be given later.

69. We close this discussion by describing a mode of submitting Helmholtz's general theory of musical quality to a further and very severe test.

The sounds of tuning-forks mounted on their appropriate resonance-boxes are, as we know, very

approximately simple tones [§ 48].   If therefore we
allow a number of such sounds, coinciding in pitch
with the fundamental-tone and individual overtones
of one and the same clang, to be simultaneously
produced, the effect on the ear ought, if Helmholtz's
theory is true, to be that of a single musical sound,
not that of a group of separate notes.   To try
the experiment in its simplest shape, take two
mounted forks forming the interval of an Octave,
and cause them to utter their respective tones to-
gether.   For a short time we are able to distinguish
the two notes as coming from separate instruments,
but soon they blend into one sound, to which we
assign the *pitch* of the *lower* fork, and a *quality*
more brilliant than that of either.   So strong is the
illusion, that we can hardly believe the higher fork
to be really still contributing its note, until we
ascertain that placing a finger on its prongs at once
changes the *timbre*, by reducing it to the dull, un-
interesting quality of a simple tone.   The character
of a clang consisting of only one overtone and the
fundamental may be shown to admit of many dif-
ferent shades of quality by suitably varying the
relative intensities of the two fork-tones in this
experiment.   If we add a fork a Fifth above the
higher of the first two, and therefore yielding the
third partial-tone of the clang of which they form

the first and second, the three tones blend as perfectly as the two did before, the only difference perceptible being an additional increase of brilliancy. The experiment admits of being carried further with the same result.

If we were able to produce by means of tuning-forks as many simple tones of the series on p. 86 as we pleased, and also to control at will their relative intensities, it would be possible to imitate in this manner the varying *timbre* of every musical instrument. The unmanageable character of very high forks has as yet prevented this being done for sounds containing a large number of powerful overtones, but an apparatus on this principle has been devised by Professor Helmholtz which imitates successfully sounds not involving more than the first six or eight partial-tones. His theory of quality is thus experimentally demonstrated both analytically and synthetically. We will examine in the next chapter certain theoretical considerations which have an important bearing on that theory.

CHAPTER VI.

ON THE CONNEXION BETWEEN QUALITY AND MODE OF VIBRATION.

70. It was stated on p. 74 that, when a pendulum performs oscillations whose extent is small compared to the length of the pendulum, *the period of a vibration is the same for any extent of swing within this limit.* We will apply this fact to prove that the prongs of a tuning-fork vibrate in the same *mode* [§ 11] as a pendulum.

When a sustained simple tone is being transmitted by the air, we may regard it as originated by a tuning-fork of appropriate pitch and size. But we know experimentally that, by suitable bowing, we may elicit from such a fork tones of various degrees of intensity, but having all the same pitch. Here, therefore, though the *extent of vibration varies,* the *period remains constant,* which is the pendulum-law. Accordingly the vibrations of a tuning-fork are identical in *mode* with those of a pendulum. The same thing will hold good of the aerial vibra-

tions to which those of a fork give rise. Hence a simple tone is due to vibrations executed according to the pendulum-law.

Such vibrations will, therefore, give rise to waves of condensation and rarefaction whose associated wave-form is that drawn in Fig. 17 *bis*, p. 40. It will be convenient to call the vibrations to which a simple tone is due *simple vibrations;* and the associated waves *simple waves.* We proceed to examine the modes of vibration corresponding to composite sounds.

Let us first take the case of a clang consisting of but *two* simple tones, the fundamental and its first overtone. A particle of air engaged in transmitting this sound is simultaneously acted upon by two sets of forces which, if they acted separately, would cause it to perform two simple vibrations whose periods were to each other in the ratio of 1 to 2. We have to investigate what its motion will be under the joint action of these forces. The problem before us is the *composition of two vibrations executed in the same straight line.*

Suppose that a series of particles originally at rest in the dotted line through *O*, Fig. 44, and capable of vibrating in paths *perpendicular to it,* are under simultaneous forces which, if acting separately, would give rise to the transmission of the parts of waves drawn in the figure. Let *a* and *b* be the

respective positions which a particle originally at O would at a given moment occupy if the forces acted separately.

71.   In determining the corresponding position of this particle under the forces acting simultaneously we employ the following principle :—*The resultant displacement is equal to the sum or difference of the component displacements, according as these are produced in the same or in opposite directions.*

Fig. 44.
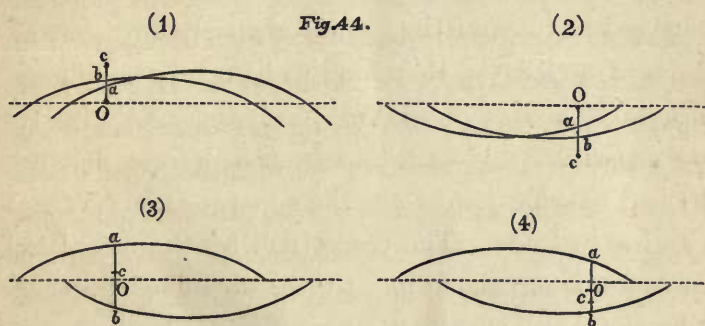
In the application of this principle four different cases present themselves.   In (1), crest falls on crest; in (2), trough on trough, and the displacement, Oc, of the particle from its position of rest, O, is in both these instances equal to the *sum* of the displacements Oa and Ob.   In (3) and (4), where the crest of one wave meets the trough of another, Oc is equal to the *difference* between Oa and Ob; c being above or below the undisturbed line, according

as *Oa* is greater than *Ob*, as in (3), or less than *Ob*, as in (4). Thus, each tributary set of forces produces its full effect in its own direction, or, in other words, the displacement due to one set is superposed on that due to the other.

In order to determine the form of the resultant wave, we have only to apply this principle to successive particles. We thus obtain an assemblage of points constituting the wave required.

The problem of the motion of an air-particle engaged in transmitting simultaneously two partial-tones of a clang is, therefore, to be solved as follows. Let each simple vibration be represented by its associated wave. Ascertain by the process just described to what joint form the superposition of these two waves leads. The result will be the associated wave-form corresponding to the mode of particle-vibration to which the compound sound is due.

72. Before, however, we can lay down the two tributary waves, an important point remains to be settled. We will, for a moment, suppose that the two simple tones on which we are engaged are sustained by two tuning-forks situated as in the annexed figure, and that we are examining the transmission of their resulting clang along the dotted line with respect to which they are symmetrically situated.

Let the forks have been set sounding by precisely simultaneous blows. They will then commence swinging out of their positions of rest *in the same*

Fig. 45.

c′

c
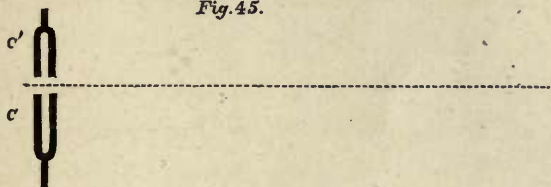
direction at the same instant. The points in an associated wave-form which correspond to the initial position of a vibrating particle are those in which the wave-form cuts the initial line. Hence in laying down the tributary waves we must, in the present instance, begin them at one and the same point in the initial line, and take care that their convexities at that point are turned the same way, as at *O*, Fig. 46.

Fig. 46.

O
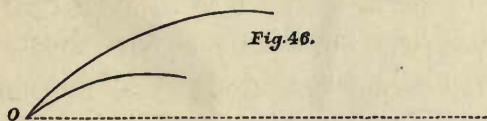
In this case the two vibrations are said to start in the *same phase.*

If the two forks are set in vibration at *different* moments, they may *not* swing out of their equilibrium positions in the same direction together. Hence we no longer necessarily have a point where *both* sets of associated waves cut the initial line. The result is

of the kind shown in Fig. 47, where three different cases are represented.

Fig. 47.
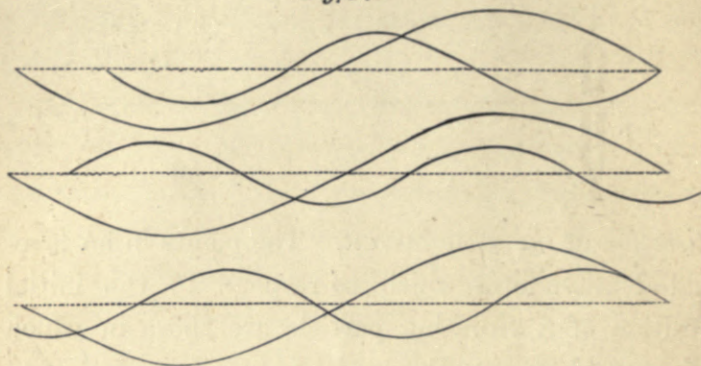
We have here vibrations starting in *different phases.*

It is clear from the figure that all phase-differences can be properly represented by merely causing the waves engaged to assume different positions upon the initial line *with reference to each other.* The second and third cases are obtainable from the first by sliding the system of shorter waves bodily along the initial line, while the other system of waves retains its position.

By the help of the instrument mentioned on p. 138 Professor Helmholtz has demonstrated that, when a number of partial-tones are independently produced, the clang into which they coalesce *has the same quality, whatever differences of phase may exist*

among the systems of simple vibrations to which the constituent partial-tones are due. Accordingly, we may expect to find that not *one single wave-form,* but *many such forms,* correspond to a sound of given quality and pitch.

In Figs. 48, 49, 50, the associated wave-form corresponding to our clang of two partial-tones (p. 140) is constructed for three degrees of phase-difference. The simple constituent waves are shown in thin, the result of their composition in full lines. In each case two complete wave-lengths of the latter are exhibited.

Figs. 51 and 52 present two wave-forms drawn, in the same way, for a clang of constant pitch and quality containing the partial-tones 1, 2 and 3.

The dissimilarity of form, and therefore of corresponding particle-vibration, is in both sets of figures very marked.

73. It has been shown that, by mere alteration of phase, a very great variety of resultant wave-forms can be obtained from two sets of simple waves of given lengths and amplitudes. *Each one of these forms* will give rise to a cycle of others, if we allow the relative amplitudes of the constituent systems to be changed, while keeping the difference of phase constant. If, therefore, we have at our disposal the systems of simple waves corresponding to an unlimited

Fig. 48.
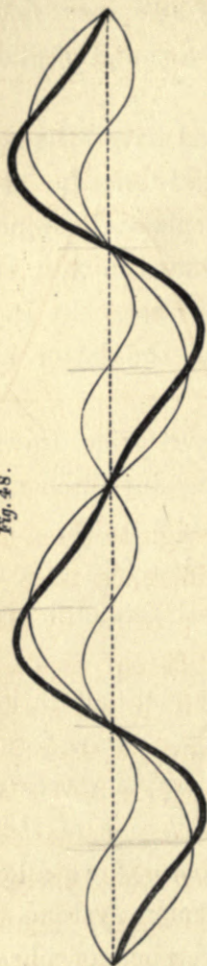
Fig. 49.

Fig. 50.
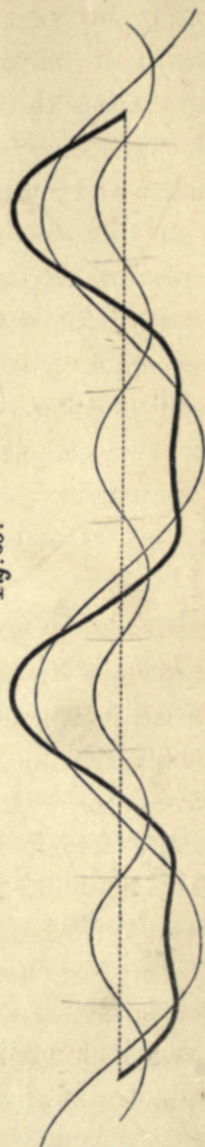
Fig. 51.
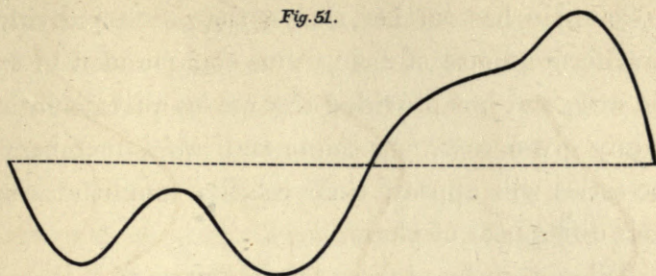
Fig. 52.
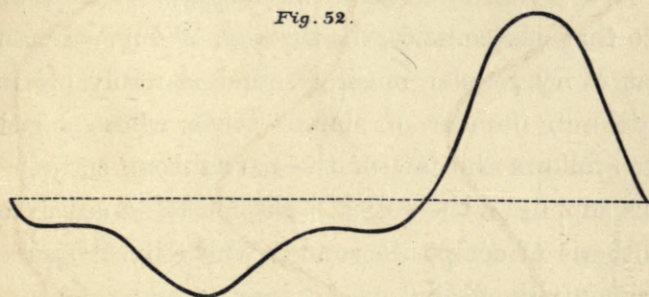
number of partial-tones, and can assign to them any degrees of intensity and phase-difference that we choose, it is manifest that we may produce by their combination an endless series of different resultant wave-forms. On the other hand, it is *not evident* that, even out of this rich abundance of materials, we could build up *every form of wave which could possibly be assigned.* The French mathematician *Fourier* has, however, demonstrated that there is no form of wave which (unless itself simple) cannot be compounded out of a number of simple waves whose lengths[1] are inversely as the numbers 1, 2, 3,

[1] Arranged in diminishing order.

4, &c.   He has further shown that each individual wave-form admits of being thus compounded in *only one way*, and has provided the means of calculating, in any given case, *how many* and *which* members of the series will appear, their relative amplitudes and their differences of phase.

When translated from the language of Mechanics into that of Acoustics, the theorem of *Fourier* asserts that every regular musical sound is resolvable into a definite number of simple tones whose relative pitch follows the law of the partial-tone series.   It thus supplies a theoretical basis for the analysis and synthesis of composite sounds which have been experimentally effected in chapters IV. and V.

When we are listening to a sustained clang, the air at any one point within the orifice of the ear can have only one definite mode of particle-vibration at any one moment.   How does the ear behave towards any such given vibration?   It proceeds as follows. If the vibration is simple, it leaves it alone.   If composite, it analyses it into a series of simple vibrations whose rates are once, twice, three times &c. that of the given vibration, in accordance with Fourier's theorem.   In the former event the ear perceives only a simple tone.   In the latter, it is able to recognise, by suitably directed and assisted efforts, partial-tones corresponding to the rate of each constituent into

which it has analysed the composite vibration origi-
nally presented to it. The ear being deaf to differ-
ences of *phase* in partial-tones (p. 144), perceives no
difference between such modes of vibration as those
exhibited on p. 146, but merely resolves them into
the same single pair of partial-tones. Since, however,
only *one* such resolution of a given vibration-mode is
possible, the ear can *never vary* in the group of
partial-tones into which it resolves an assigned
clang.

The power possessed by the ear of thus singling
out the constituent tones of a clang and assigning
to them their relative intensities is unlike any cor-
responding capacity of the eye. Take for instance
the two curves shown in Figs. 51 and 52, and try
to determine, by the eye alone, what simple waves,
present with what amplitudes, must be superposed
in order to reproduce those forms. The eye will be
found absolutely to break down in the attempt.

We have seen that the *loudness* of a composite
sound depends on *extent* of vibration, and its *pitch*
on *rate* of vibration. There remains only one variable
element, viz. *mode* of vibration, to account for the
quality of the sound. From this consideration it
follows that *some connexion* must exist between the
quality of a sound and the mode of aerial vibration
to which the sound is due. Up to the time of Helm-

holtz no advance had been made in clearing up the *nature* of this connexion.   It was reserved for him to show that, while no two sounds of different quality can correspond to the same mode of vibration, an indefinitely large number of modes of vibration may give rise to a sound of only one degree of quality. In other words, mode of vibration determines quality, but quality does not determine mode of vibration.

CHAPTER VII.

ON THE INTERFERENCE OF SOUND, AND ON 'BEATS'.

74. In § 71 we examined the principle on which the general problem of the composition of vibrations is solved. We now approach certain very important particular cases of that problem, which it will be worth while to solve both independently and as instances of the method repeatedly applied in § 72.

Suppose that a particle of air is vibrating between the extreme positions *A* and *B* while convey-

Fig.53.

A   *B*
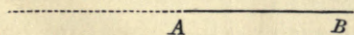
ing a sustained simple tone produced by a tuning-fork, or stopped flue-pipe. Now let a second instrument of the same kind be caused to emit a tone *exactly in unison with the first*. We will assume that, when the waves of the second tone reach the particle, it is just on the point of starting from *A* towards *B*. Two extreme cases are now possible, depending on the movement which the particle would have executed in virtue of the later-impressed vibration alone.

First, suppose *that* to be from A along the line AB, either through a greater or less distance than AB, back again to A, and so on.   Here the separate effects of the two sets of vibrations will be *added together*, the particle will, therefore, perform vibrations of *larger extent* than it would under either component separately.   Next, suppose that, under the second set of vibrations alone, the particle would move from A in the opposite direction to its former course, *i.e.* along BA produced, shown by a dotted line in the figure. In this case the separate effects are absolutely antagonistic ; accordingly the joint result is that due to the *difference* of its components.   The particle will, therefore, execute *less extensive* vibrations than it would have done under the more powerful of the two components acting alone.

The most striking result presents itself when the two systems of vibrations, besides being in opposition to each other, are also exactly equal in extent.   In this case the air-particle, being solicited by equal forces in opposite directions, remains at rest, the two systems of vibrations completely neutralising each other's effects.   In general, however, these systems, even when equal in extent of vibration, are neither in complete opposition nor in complete accordance, but in an intermediate attitude, so as only partially to counteract, or support, each other.   These conclusions

admit of being exhibited in a more complete manner by means of associated waves. We have only to lay down the simple wave-forms corresponding to the constituent vibrations, and superpose them as in § 72. The reader will have noticed that the differences of relative motion described on the preceding page are merely phase-differences.

Fig. 54.

(1)

(2)

(3)
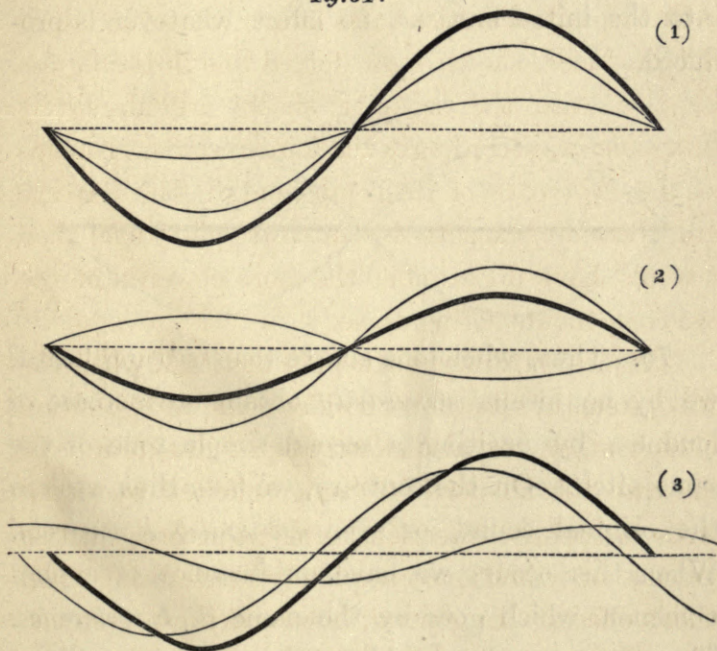
In Fig. 54, (1), (2), (3), we have two waves of unequal amplitudes in complete accordance, complete antagonism and an intermediate condition, respec-

tively. In Fig. 55, a case of equal and opposite waves is shown. In (1) Fig. 54, the resultant wave is the *sum,* and in (2) the *difference* of the component waves. In (3), we get a wave of intermediate amplitude. These three resultant waves are necessarily *simple,* as otherwise two simple tones in unison would give rise to a composite sound, which would be absurd. In Fig 55 the wave-form degenerates into the initial line, *i.e.* no effect whatever is produced.

Fig. 55.

75. Thus, when one simple tone is being heard, we by no means *necessarily* obtain an increase of loudness by exciting a second simple tone of the same pitch. On the contrary, we *may* thus weaken the original sound, or even *extinguish it entirely.* When this occurs we have an instance of a phenomenon which goes by the name of *Interference.* That *two sounds should produce absolute silence* seems, at first sight, as absurd as that two loaves should be equivalent to no bread. This is, however, only because we are accustomed to think of Sound as

something possessing an external substantial exis-
tence ; not as consisting merely in a *state of motion of
certain air-particles,* and therefore liable, on the
application of suitable forces, to be absolutely anni-
hilated.

A single tuning-fork presents an example of this
very important phenomenon. Each prong sets up
vibrations corresponding to a simple tone, and the
two tones so produced are of the same pitch and
intensity. If the fork, after being struck, is held
between the finger and thumb, and made to re-
volve slowly about its own axis, four positions of
the fork with reference to the ear will be found
where the sound goes completely out. These posi-
tions are mid-way between the four in which the
plane sides of the prongs are held straight before the
ear. As the fork revolves from one of these positions
of loud sound to that at right-angles to it, the
sound gradually wanes, is extinguished in passing
the Interference-position, reappears very feebly im-
mediately afterwards, and then continues to gain
strength until the quarter of a revolution has been
completed.

76. The case of coexistent unisons has now been
adequately examined : we proceed to enquire what
happens when two simple tones *differing slightly in
pitch* are simultaneously produced. The problem is,

in fact, to compound two sets of pendulum-vibrations
whose periods are not exactly equal.   Let us fix on a
moment of time at which the two component vibra-
tions are in complete accordance, and suppose that a
resultant vibration is just commencing from left to
right.   It will be convenient to call this an *outward*,
and its opposite an *inward* swing.   Since the periods
of the two component vibrations are unequal, one of
them will at once begin to gain on the other, and
therefore, directly after the start, they will cease to be
in complete accordance.   It is easy to ascertain what
their subsequent bearing towards each other will be,
by considering two ordinary pendulums of unequal
periods, both beginning an outward swing at the
same instant.   Let *A* be the slower, *B* the quicker
pendulum.   When *A* has just finished its outward
swing, *B* will have already turned back and per-
formed a portion of its next inward swing.   Thus,
during each successive swing of *A*, *B* will gain a
certain distance upon it.   When *B* has, in this
manner, gained one whole swing, *i.e.* half a complete
oscillation, upon *A*, it will begin an inward swing *at
the moment when A is commencing an outward swing.*
The two vibrations are here, for the moment, in *com-
plete opposition.*   After another interval of equal
length, *B*, having gained another whole swing, will
be one complete oscillation ahead of *A*, and they will

therefore start on the next outward swing together, *i.e.* the vibrations will be momentarily in *complete accordance.* Thus, during the time requisite to enable *B* to perform one entire oscillation more than *A*, there occur the following changes. Complete accordance of vibrations, lasting only for a single swing of the more rapid pendulum, followed by partial accordance, in its turn gradually giving way to discordance, which culminates in complete opposition at the middle of the period, and then, during its latter half, gradually yields to returning accordance, which regains completeness just as the period closes.

It follows from this that, in the case of two simple tones differing slightly in pitch, we must hear a sound going through regularly recurring alternations of loudness in equal successive intervals of time, its greatest intensity exceeding, and its least intensity falling short of, that of the louder of the two tones. Each recurrence of the maximum intensity is called a *beat,* and it is clear that exactly *one* such beat will be heard in each interval of time during which the acuter of the two simple tones performs *one* more vibration than the graver tone. Accordingly, the number of beats heard in any assigned time will be equal to the number of complete vibrations which the one tone gains on the

other in that time. We may express this result more briefly as follows :—

The number of beats per second due to two simple tones is equal to the difference of their respective vibration-numbers.

77.   By means of the associated wave-forms we can obtain a graphic representation of beats, which will probably be more directly intelligible than any verbal description. In Fig. 56, the constituent simple waves are laid down, and their resultant is constructed, for the interval of a semi-Tone. The vibration-fraction for that interval is $\frac{16}{15}$; *i.e.* 16 vibrations of the higher tone are performed in the same time as 15 of the lower. The figure represents completely the state of things from a *maximum* of intensity to the adjacent *minimum*. The time during which this change occurs is one-half of that above-mentioned : accordingly the figure shows 8 and 7½ wave-lengths of the respective systems. Thus *half a beat* is here pictorially represented, the amplitude of the resultant waves steadily diminishing during this period. We have only to turn the figure upside down, to get a picture of what occurs in the next following equal period. The amplitudes here again increase until they regain their former dimensions. *One whole beat* is thus accounted for.
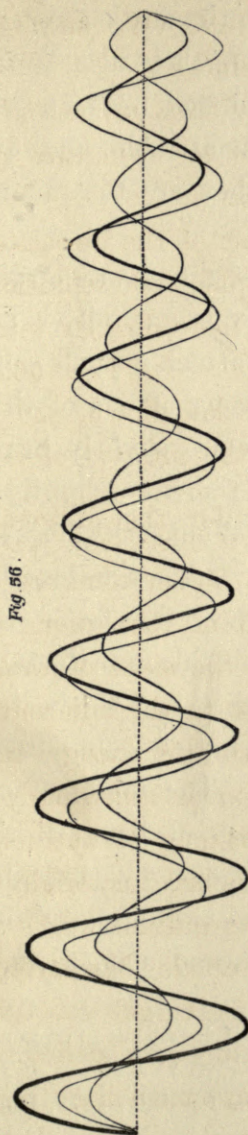
Fig 56.

In addition to the alternations of intensity which characterise beats, they also contain variations of pitch. The existence of such variations is both theoretically demonstrable and experimentally recognisable, but they are too minute to require examination here[1].

78.   The most direct way of studying the beats of simple tones experimentally is to take two unison tuning-forks and attach a small pellet of wax to the extremity of a prong of one of them. The fork so operated on becomes slightly heavier than before; its vibrations are therefore slightly retarded, and its pitch lowered. When *both* forks are struck and held to the ear, beats are heard. These will be most distinct when the forks' tones are exactly equally loud, for in this case the minima of intensity will be equal to zero, and the beats will therefore be separated by intervals of absolute, though but momentary, silence. The increase in rapidity of the beats, as the interval between the beating tones widens, may be shown by gradually loading one of the forks more and more heavily with wax pellets, or by a small coin pressed upon them. If it is desired

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[1] The reader may, if he wishes to pursue this subject, refer to a Paper, by the author, on ' *Variations of Pitch in Beats,*' in the *Philosophical Magazine* for July, 1872, from which Fig. 56 has been copied.

to exhibit these phenomena to several persons at once, the forks should first be mounted on their resonance-boxes, and, after the pellets have been attached, stroked with the resined bow, care being taken to produce tones as nearly as possible equal in intensity. Slow beats may also be obtained from any instrument capable of producing tones whose vibration-numbers differ by a sufficiently small amount. Thus, if the strings corresponding to a single note of the pianoforte are not strictly in unison, such beats are heard on striking the note. If the tuning is perfect, a wax pellet attached to one of the wires will lower its pitch sufficiently to produce the desired effect. Beats not too fast to be readily counted arise between adjacent low notes on the harmonium, or, still more conspicuously, on large organs. They are also frequently to be heard in the sounds of church bells, or in those emitted by the telegraph wires when vibrating in a strong wind. In order to observe them in the last instance, it is best to press one ear against a telegraph-post and close the other : the beats then come out with remarkable distinctness. It should be noticed that, when we are dealing with two composite sounds, several sets of beats may be heard at the same time, if pairs of partial-tones are in relative positions suited to produce them. Thus, suppose that two

clangs coexist, each of which contains the first six
partial-tones of the series audibly developed.   Since
the second, third, &c. partial-tones of each clang make
twice, three times &c. as many vibrations per second
as their respective fundamental-tones, [§ 43] it follows
that the differences between the vibration-numbers
of successive pairs of partial-tones belonging to the
two clangs will be twice, three times, &c., the differ-
ence between the vibration-numbers of the two
fundamental-tones.   Accordingly, if the fundamental-
tones give rise to beats, we may hear, in addition to
the series so accounted for, five other sets of beats,
respectively twice, three, four, five and six times as
rapid as they.   In order to determine the number of
beats per second for any such set, we need only mul-
tiply the number of the fundamental beats by the
order of the partial-tones concerned.   The beats of
two simple tones necessarily become more rapid if
the higher tone be sharpened, or the lower flat-
tened ; *i.e.* if the interval they form with each other
be widened.   The beats may, however, also be ac-
celerated without altering the interval, by merely
placing it in a higher part of the scale.   Greater
vibration-numbers are thus obtained with a pro-
portionately larger difference between them, though
their ratio to each other remains what it was be-
fore.   Thus the rapidity of the beats due to an

assigned interval depends jointly on two circum-
stances, the width of the interval and its position
in the musical scale ; in other words, on both the
relative and *absolute* pitch of the tones which
form it.

CHAPTER VIII.

79. A question of fundamental importance now presents itself, viz. *What becomes of beats, when they are so rapid that they can no longer be separately perceived by the ear?* In order to answer it, the best plan is to take two unison-forks of medium pitch, mounted on their resonance-boxes, attach a small pellet of wax to a prong of one of them, and then gradually increase the quantity of wax. At first very slow beats are heard, and as long as their number does not exceed four or five in a second, the ear can follow and count them without difficulty. As they become more rapid the difficulty of counting them augments, until at last they cease to be separately recognisable. Even then, however, the ear retains the conviction that the sound it hears is a series of rapid alternations, and not a continuous tone. Its intermittent *character* is not lost, although the intermittances themselves pass by too rapidly

for individual recognition.   Exactly the same thing may be observed in the *roll* of a side drum, which no one is in danger of mistaking for a continuous sound.

Rapid beats produce a decidedly harsh and grating effect on the ear; and this is quite what the analogy of our other senses would lead us to expect.   The disagreeable impressions excited in the organs of sight by a flickering unsteady light, and in those of touch by tickling or scratching, are familiar to every one.   The effect of rapid beats is, in fact, identical with the sensation to which we commonly attach the name of *dissonance*.   Let us examine, in somewhat greater detail, the conditions necessary for its production between two simple tones.   If we take a pair of middle-*C* forks, and gradually throw them more and more out of unison with each other in the way already described, the roughness due to their beats reaches its maximum when the interval between them is about a half-Tone: for a whole Tone it is decidedly less marked, and when the interval amounts to a Minor Third, scarcely a trace of it remains.   Hence, in order that dissonance may arise between two simple tones, they must form with each other a narrower interval than a Minor Third.   If we call this interval the *beating-distance* for two such tones, we may express the above condition thus. Dissonance can arise directly between two simple

tones *only when they are within beating-distance of
each other*[1].

It follows from what was shown at the end of
Chapter VII that the same beating interval will
give rise to very different numbers of beats per
second according as it occupies a low or high position
in the scale. Such an interval, *e.g.* a whole Tone,
becomes palpably less dissonant as it is successively
raised in pitch. Accordingly the beating-distance
which for tones of medium pitch we have roughly
fixed at a *Minor Third*, must be supposed to expand
somewhat in low and correspondingly contract in
high regions of the scale.

The general partial-tone series consists of simple
tones which, *up to the seventh*, are mutually out of
beating-distance : above the seventh they close in
rapidly upon each other. In the neighbourhood of
10, the interval between adjacent partial-tones is
about a whole Tone ; near 16, a semi-Tone ; higher
in the series they come to still closer quarters.
These high partial-tones are therefore so situated as
to produce harsh dissonances with each other.
Where they are strongly developed in a clang, there

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[1] It will be shown in the sequel [§§ 89—92] that dissonance
may, in the case of simple tones forming intervals wider than a
Minor Third, arise from beats other than those here under con-
sideration.

will therefore be a certain inevitable roughness in its *timbre*. This is the cause of the harsh quality of trumpet or trombone notes, and also of the shrill tremulous sounds sometimes observed in the human voice (p. 136). In fact we may regard the partial-tone series above the seventh tone, when fully repre-sented, as contributing mere noise to the clang. This explains why it is advantageous that only the lower partial-tones of organ reeds should be intensi-fied by resonance [see § 66, p. 132].

80. It has been shown that, when two simple tones are simultaneously sustained, beats can arise directly between them only under one condition, viz. that the tones shall differ in pitch by less than a Minor Third, or thereabouts. When, however, the two co-existing sounds are no longer simple tones, but composite clangs each consisting of a series of well-developed partial-tones, the case becomes alto-gether different. Let us examine the state of things which then presents itself.

The sounds of most musical instruments practi-cally contain only the first six partial-tones; we will, therefore, assume this to be the case with the clangs before us. No beats can then arise between partial-tones of the *same clang* for the reason as-signed on p. 166. Dissonance due to beats will, however, be produced if a partial-tone belonging

to one clang is within the specified distance of a partial-tone belonging to the other clang. Several pairs of tones may be thus situated, and, if so, each will contribute its share of roughness to the general effect. The intensity of the roughness due to any such pair will depend chiefly on the respective orders to which the beating partial-tones belong, and on the interval between them. The lowest partial-tones being the loudest, produce the most powerful beats, and half-Tone beats are, in general, harsher than those of a whole Tone. In determining the general effect of a combination of two clangs, we have to ascertain what pairs of partial-tones come within beating-distance, and to estimate the amount of roughness due to each pair. The joint effect of all these roughnesses, if there are several such pairs, or the roughness of a single pair if there be but one, constitutes the *dissonance of the combination.* If there be *no dissonance,* the combination is described as a *perfect concord.* When dissonance is present, its *amount* will decide whether the combination shall be called an *imperfect concord* or a *discord.* The line separating the two must therefore, of necessity, be somewhat arbitrarily drawn.

81. Let us examine the principal consonant intervals, in the manner above described, beginning with the Octave.
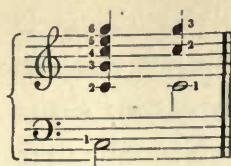
The minims here represent the fundamental-tones ; the crotchets above them corresponding over-tones. Those belonging to the higher clang are only written down as far as the third, since the fourth, fifth and sixth meet no corresponding tones of the lower clang with which to form beating pairs. As long as the tuning is perfect each partial-tone of the higher clang coincides exactly with one belonging to the lower. No dissonance can therefore occur, and the combination is a perfect concord. But suppose the higher C to be out of tune : each of its partial-tones will be correspondingly too sharp or too flat, and three sets of beats will be heard between the partial-tones 2—1, 4—2 and 6—3. When the higher C is as much as a semi-Tone wrong, the result is

The pair 2—1 is of the most importance, and gives in each case *sixteen beats per second*. The two

others give respectively 32 and 48 beats per second. A semi-Tone corresponds to about maximum roughness in the middle region of the scale, so that we have before us an exceedingly harsh discord. As the pitch of the higher note is gradually corrected, the rapidity of the beats diminishes, but the tuning must be rigorously accurate to make them entirely vanish. If the note makes but *one vibration per second* too many or too few, which corresponds to a difference in pitch of only about a *thirtieth part of a whole Tone,* the defect of tuning makes itself felt by three sets of beats, of 1, 2 and 3 per second respectively. The tuner must keep slightly altering the pitch until he at length hits on that which completely extinguishes the beats. We saw in an earlier part of this inquiry (p. 68) that, when two sounds form with each other the interval of an Octave, their vibration-numbers must be in the ratio of $2 : 1$. Long after it had been experimentally ascertained that rigorous compliance with this arithmetical condition was essential for securing a perfectly smooth Octave, the reason for this necessity remained entirely unknown, and nothing but the vaguest and most fanciful suggestions were offered to account for it—such as, for instance, that "the human mind delights in simple numerical relations." This attempt at explanation overlooked the obvious

fact that many people who knew nothing either about vibrations or the delights of simple numerical relations, could tell a perfect Octave from an imperfect one a great deal better than most men of science. The true explanation, which it was left for Helmholtz to discover, lies in the fact that *only by exactly satisfying the assigned numerical relation can the partial-tones of the higher clang be brought into exact coincidence with partial-tones of the lower, and thus all beats and consequent dissonance prevented.*

82. No narrower interval than an Octave can be found which gives an *absolutely* perfect concord. The nearest approach to such a concord is the Fifth :—
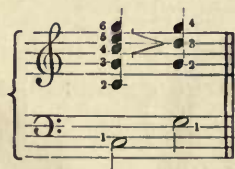
Here we get two pairs of coincidences 3—2 and 6—4, but a certain roughness is caused by 3 of the higher clang being within beating-distance of both 4 and 5 of the lower clang. It is true that, since 4 and 5 are generally weak, and the beating intervals whole Tones, this roughness will be but slight : still the dissonance thus caused prevents our classing the Fifth as an interval quite equally smooth with the

Octave. The tuning must be perfectly accurate,
the Fifth being closely bounded by harsh discords.
The result of an error of a semi-Tone is as follows :—
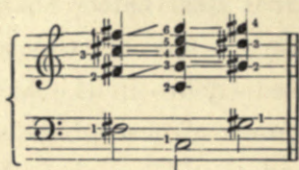
For every single vibration per second by which
the higher clang is out, there will be two beats per
second from the pair 3—2, with others of greater
rapidity, but less intensity, from the higher pairs.

83.   For the Fourth we have
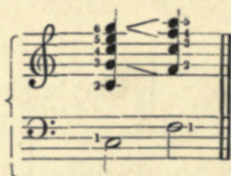
The amount of dissonance is greater than in the
case of the Fifth, since 3 and 2 are usually *both*
powerful tones, and therefore produce louder beats
than those of 4—3 and 5—3.   There are, in addi-
tion, the beating pairs 5—4, 6—4 and 6—5.   More-
over the first pair of coincident partial-tones, 4—3,
are in general *weaker* than the beating pair below
them, 3—2.   The Fourth is bounded only on one
side by a harsh discord.   If its upper clang is half a

note too sharp, we have the interval $C—F\sharp$, which is treated in the last figure but one.   Slight flattening of the F will set the pair 4—3 beating slowly ; the disappearance of their beats thus secures the accurate tuning of the interval.   On the other hand, lowering F weakens the beats of 3—2 by widening the distance between those tones, and therefore tends to lessen the whole amount of roughness.   These considerations go far to explain the fact that a long dispute runs through the history of Music, as to whether the Fourth ought to be treated as a concord or as a discord.   The decision ultimately arrived at in favour of the first of these alternatives was perhaps, as Helmholtz suggests, due more to the fact that the Fourth is the inversion[1] of the Fifth than to the inherent smoothness of the former interval.

84.   Next come the intervals of the Major Third and Major Sixth, which shall be taken together, as they are very nearly equally consonant.
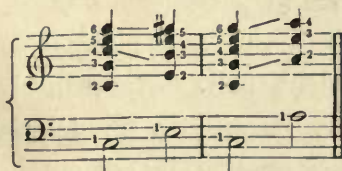
The dissonance due to the pair 3—2, separated by a Tone in the Sixth, is perhaps about equal to

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[1] See § 97 *post.*

that of the weaker pair 4—3, which are only a semi-
Tone apart in the Third. The 'definition' of these
intervals, *i.e.* the accuracy of tuning requisite for a
good effect, will, as it depends in each on the *fifth*
partial-tone of the lower clang, be in general not great.

85. The remaining intervals narrower than an
Octave which rank as concords are the Minor Third
and Minor Sixth.

Each contains strong elements of dissonance ; in
fact we are here near the boundary between concords
and discords. As regards sharpness of definition, the
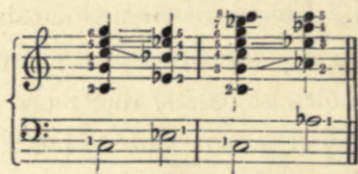
tones 6 and 5, on which it depends in the first of the
two intervals, are, in the sounds of many instruments,
weak or even entirely absent, while for the second
interval the series of partial-tones must be extended
as far as the 8th of the lower clang in order to reach
the first coincident pair. Accordingly the Minor
Sixth can hardly be said to be defined *at all*, for
clangs of ordinary quality, by coincidence of partial-
tones. Its powerful beating pair 3—2, separated by
the interval of greatest dissonance, a semi-Tone,
makes it the roughest of all the concords. On the

pianoforte, and other instruments of fixed sounds, *the same notes ($CA^\flat$) which represent the Minor Sixth have also to do duty as one of the harshest discords, the Sharp Fifth, ($CG\sharp$).* The extremely defective consonance of the Minor Sixth could hardly be more conclusively shown than by the fact just mentioned.

86.   As regards the dissonant intervals of the scale, we have, in addition to those incidentally examined above, the semi-Tone, Tone and Minor Seventh.   The first two need not be examined, since obviously each pair of corresponding overtones are brought within the same beating intervals as the two fundamentals.   The dissonance resulting is, of course, harsher for the half than for the whole Tone. The Minor Seventh is constituted thus :—
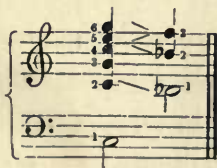
It is the mildest of the discords, in fact in mere smoothness it decidedly surpasses the Minor Sixth.

87.   In order that the reader may see at a glance the whole result of this somewhat laborious discussion, we subjoin a graphical representation of the amount of dissonance contained in the several intervals of the scale.   The figure is taken, with some slight altera-

tions, from one given in Helmholtz's work. The intervals, reckoned from middle-*C*, are denoted by distances measured along the horizontal straight line. The dissonance for each interval is represented by the vertical distance of the curved line from the corresponding point on the horizontal line. The calculations on which the curve is based were made by Helmholtz for two constituent clangs of the quality of the violin.

Fig. 57.
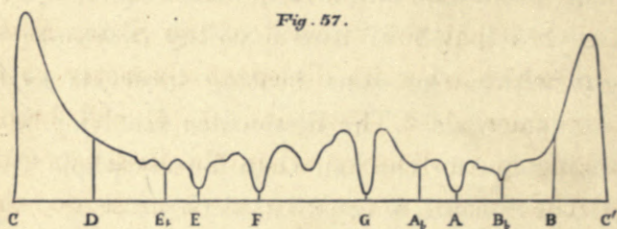
The figure indicates the sharpness of definition of an interval by the steepness with which the curve ascends in its vicinity. If we regarded the outline as that of a mountain-chain, the discords would be represented by *peaks*, and the concords by *passes*. The lowness and narrowness of a particular pass would measure the smoothness and definition of the corresponding musical interval.

88. The theory of musical consonance and dissonance, our examination of which is now concluded, shows that the degree of smoothness or roughness

possessed by an assigned interval is not fixed and invariable, but *dependent on the quality of the sounds by which the interval is held.* The results we have arrived at are generally true for sounds containing the first six partial-tones, but they will not apply, without modification, to clangs differently constituted. Let us take a case or two in illustration of this point. Suppose, for instance, we are dealing with sounds such as those of stopped organ-pipes which contain *only odd partial-tones* (p. 124). It is at once clear from p. 172 that the interval of the Sharp Fourth $C—F\sharp$, which owes its dissonant character to the beating pairs 3—2, 4—3 and 6—4, will become something quite different when the dissonance due to all these pairs disappears, as it must do, since each of them contains at least *one* partial-tone of an *even* order. The Minor Sixth would also gain in smoothness in such a *timbre*, by the removal of the loudly discordant pair 3—2.

Further, if the two notes forming any interval are held by instruments of different quality, the smoothness of the combination may greatly depend on which instrument takes the lower and which the upper note. Thus *e.g.* for the interval of a Fifth, reference to p. 171 will show that the instrument which has the weaker *third partial-tone* should for the smoother effect take the upper note, since that

arrangement reduces to a minimum the dissonance due to the pairs 4—3 and 5—3. For the interval of a Fourth, the opposite arrangement gives the smoothest effect by minimising the dissonance 3—2. Experiments confirming theoretical conclusions of this kind may be made with great ease on the stops of an organ possessing more than a single manual.

The above considerations laid down by Helmholtz open a whole field of results which I believe to be entirely in advance of any hitherto obtained by Musical theorists[1].

89. It is possible to draw from the general theory of consonance and dissonance an inference which seems, at first sight, fatal to the truth of the theory itself. "If," it may be said, "the difference between a consonant and a dissonant interval depends entirely on the behaviour towards each other of certain pairs of overtones; then, in the case of sounds like those of large stopped flue-pipes, *where there are no overtones at all*, the distinction between concords and discords ought entirely to disappear, and the interval of a Seventh, for instance, to sound just as smooth as that of an Octave. As this is not the fact, the theory cannot be true."

[1] "Musical *theory*" is as misleading a term as could possibly be invented. "Musical *empiricism*" would replace it with conspicuous advantage.

In order to meet this objection, it will be necessary first to acquaint the reader with certain known experimental facts which Helmholtz has dragged out of the obscurity in which they had lain for fully a century, and forced to deliver their testimony in completion of his theory.

90. Let two tuning-forks of different pitch mounted on their respective resonance-boxes be thrown into *powerful* vibration by a resined bow. With adequate attention it is possible to recognise, in addition to the tones of the forks themselves, certain new sounds which usually differ in pitch from *both* the former ones. These sounds, called from the manner of their production *combination-tones*, fall into two categories, with only one of which, discovered in 1740 by a German organist named *Sorge*, we need now concern ourselves. It consists of a series classed as combination-tones of the *first, second, third, &c., orders*, of which the *first* is of the most importance, as it can be heard without difficulty. Its pitch is determined by the following law. *The combination-tone of the first order of two simple primary-tones has for its vibration-number the difference between the respective vibration-numbers of the primaries.* Thus, *e.g.*, if the two primaries make 200 and 300 vibrations per second, and therefore form a Fifth with each other, the combination-tone will

12—2

make 100 vibrations per second, and therefore lie exactly one Octave below the graver of the two primary tones. By applying this rule we can determine the pitch of the first-order combination-tone for a pair of simple primaries forming any given interval with each other. The following table, copied from Helmholtz's work, shows the results for all the consonant intervals not exceeding one Octave.

Interval.	Vibration-ratio.	Difference.	Depth of the Combination-tone below the graver primary.
Octave	1 : 2	1	Unison
Fifth	2 : 3	1	Octave
Fourth	3 : 4	1	Twelfth
Major Third	4 : 5	1	Two Octaves
Minor Third	5 : 6	1	Two Octaves & a Major Third
Major Sixth	3 : 5	2	Fifth
Minor Sixth	5 : 8	3	Major Sixth.

In Musical notation the same thing stands thus, the primaries being denoted by minims and the combination-tones by crotchets.
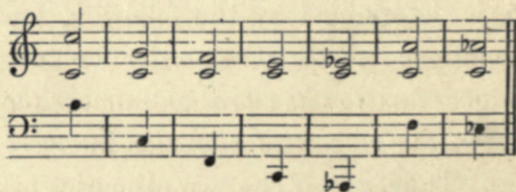
Combination-tones are produced with unusual distinctness by the harmonium. If the primaries

shown in the treble stave are played on that instrument while the pressure of air in the bellows is vigorously sustained, the corresponding combination-tones of the first order, written in the bass, come out with unmistakable clearness. With pianoforte-sounds, on the contrary, combination-tones can be recognised only when the primaries are struck very forcibly, and they are always extremely faint and rapidly evanescent. If, however, the key corresponding to the combination-tone sought be first silently pressed down, its note is sometimes sustained by resonance when the keys of the two primaries have been smartly struck '*staccato*'[1].

Combination-tones of the *second order* may be treated as if they were first-order tones produced between one or other of the primaries and the combination-tone of the first order. Similarly we may regard each combination-tone of the *third order* as due to a second-order tone paired either with one of the primaries, with the first-order tone or with its own fellow of the second order. Successive subtraction will therefore enable us to determine the vibration-number of a combination-tone of any order from the vibration-numbers of the two primaries.

Combination-tones grow rapidly feebler as their

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[1] *i.e.* allowed to rise again as quickly as possible after the blow.

order becomes higher.   Those of the first order
are usually distinct enough, and those of the second
to be heard with a little trouble.   The third order is
only recognisable when entire stillness is secured,
and the greatest attention paid.   It is a moot point
whether the fourth-order tones can be heard at all.

91.   We can now show that the existence of
combination-tones prevents intervals formed by two
simple tones from altogether lacking the characteristic
differences of consonance and dissonance, though
those differences are far less marked than in the
case of composite sounds.   To begin with the Octave.
Let us suppose that we have two simple tones form-
ing nearly this interval, but that the higher of them
is a little sharp, so that the Octave is not strictly
in tune, is what is called slightly *impure*.   Let the
lower tone make 100, the higher 201, vibrations per
second.   They will give rise to a combination-tone
making 101 vibrations per second (p. 179), and this
with the lower primary will produce *one beat per
second*.   If the higher primary had been flat instead
of sharp, making say 199 vibrations per second, we
should have had 99 as combination-tone, giving rise,
with 100, to beats of the same rapidity as before.
These beats cannot be got rid of except by making
the vibration-ratio exactly 1 : 2, *i.e.* the Octave
perfectly pure.   The roughness must increase both

when the interval widens and when it contracts, so that the Octave, in simple tones, is a well-defined concord bounded on either side by decided discords. This result may be easily verified experimentally by taking two tuning-forks forming an Octave with each other, and throwing the interval slightly out of tune by causing a pellet of wax to adhere to a prong of one of them. On vigorously exciting the forks the beats will be distinctly heard[1].

The Octave is the only interval which is defined by the beats of a combination-tone of the first order with one of the primary tones. For the next smoothest concord, that of the Fifth, we are obliged to have recourse to the second order. Thus, proceeding by successive subtraction, we have :—

Primaries	200		301
C. T. of 1st order		101	
C. Ts. of 2nd order	99		200
No. of beats per sec.		2	

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[1] As, however, this mode of treatment produces very perceptible overtones, [§ 48, *note.*] the experimenter must be on his guard against attributing to combination-tone beats an effect which is really due to beats of overtones. There is special risk of being thus led astray in the case of the Octave, where the first-order combination-tone coincides in pitch with the lower of the two primaries. The result of my own observation is that with mounted forks excited in the *gentlest possible way* there is *but slight* dissonance even in an impure Octave.

The Fifth is, thus, a fairly well-defined concord, though decidedly less sharply bounded than the Octave, owing to the feebleness of the C. T. of the second order. For the Fourth we have :—

Primaries	300		401
C. T. of 1st order		101	
C Ts. of 2nd order	199		300
C. Ts. of 3rd order	101	202	98
No. of beats per sec.		3	

The third-order tones being excessively weak, the interval of a Fourth can scarcely be said to be defined at all. Still less can the remaining consonant intervals of the scale, by the evanescent beats of still higher orders of combination-tones.

92. With two moderately loud simple tones, then, the case stands thus. The interval of a Second is rendered palpably dissonant by direct beats of the primaries; that of a Major Seventh slightly so by beats of a first-order combination-tone with one of the primaries. There is a certain amount of dissonance in intervals slightly narrower or slightly wider than a Fifth, but of a feebler kind than in the case of the Octave, inasmuch as it is due to only a second-order combination-tone. Whatever dissonance may exist near the Fourth is practically imperceptible. All other intervals are free from dissonance. Accordingly all intervals from

the Minor Third nearly up to the Fifth, and from a little above the Fifth up to the Major Seventh, ought to sound equally smooth.   This conclusion is probably very inconsistent with the views of Musical theorists, who are apt to regard concord and discord as entirely independent of quality, but it is strictly borne out by experiment.   The intervals lying between the Minor and Major Thirds, and between the Minor and Major Sixths, though sounding somewhat *strange*, are entirely free from roughness, and therefore cannot be described as dissonant.

Helmholtz advises such of his readers as have access to an organ to try the effect of playing alternately the smoothest concords and the most extreme discords which the Musical scale contains on stops yielding only simple tones, such *e.g.*, as the flute, or stopped diapason.   The vivid contrasts which such a proceeding calls out on instruments of bright *timbre*, like the pianoforte and harmonium, or the more brilliant stops of the organ such as principal, hautbois, trumpet &c., are here blurred and effaced, and everything sounds dull and inanimate in consequence. Nothing can show more decisively than such an experiment that the presence of overtones confers on Music its most characteristic charms.

Thus the remark put into the mouth of a supposed objector in § 89 turns out to be no objection

whatever to Helmholtz's theory of consonance and dissonance, but, so far as it represents actual facts, to be valid against the prevalent views of Musical theorists.

93. A point connected with combination-tones, which might otherwise occur as a difficulty to the reader's mind, shall here be briefly noticed. When two clangs coexist, combination-tones are produced between every pair which can be formed of a tone from one clang with a tone from the other. These intrusive tones will usually be very numerous, and, for aught that appears, may interfere with those originally present to such an extent as to render useless a theory based on the presence of partial-tones only. Helmholtz has removed any such apprehension by showing generally that *dissonance due to combination-tones produced between overtones never exists except where it is already present by virtue of direct action among the overtones themselves.* Thus the only effect attributable to this source is a somewhat increased roughness in all intervals except absolutely perfect concords. No modifications, therefore, have to be introduced on this score into the conclusions of §§ 81—86.

CHAPTER IX.

94. In the ensuing portion of this inquiry we shall have to make more frequent use than hitherto of vibration-fractions. It may, therefore, be well to explain at this point the rules for their employment, in order that the student may acquire the requisite facility in handling them. The vibration-fraction of an assigned interval expresses the ratio of the numbers of vibrations performed *in the same time* by the two notes which form the interval. The particular length of time chosen is a matter of absolute indifference. The upper note of an Octave, for instance, vibrates twice as often as the lower does in any time we choose to select, be it an hour, a minute, a second or a part of a second. It is often convenient to determine the vibration-fraction of an interval from the vibration-numbers of its constituent notes: in doing so we choose *one second* as our time of comparison, and in this way vibration-fractions were

defined in § 34: any other standard is however equally legitimate, though, in general, less convenient. To illustrate these remarks on a particular case, the vibration-fraction $\frac{5}{4}$ indicates that, while the lower of two notes forming a Major Third makes four vibrations, the higher of them makes five. Therefore, while the lower makes one vibration, the higher makes $\frac{5}{4}$ths of a vibration, or $1\frac{1}{4}$ vibrations. Conversely, while the higher note makes one vibration, the lower makes $\frac{4}{5}$ths of a vibration. The same reasoning being equally applicable to other cases, it follows that any fraction greater than unity may denote the number of vibrations and fractions of a vibration made by the higher of two notes forming an interval, while the lower of them is making a single vibration. Similarly, any fraction less than unity may indicate the proportion of a whole vibration performed by the lower note, while the upper is making one complete vibration.

We will next investigate rules for determining the vibration-fractions of the sum and difference of any two intervals whose separate vibration-fractions are known.

95. Suppose that, starting from a given note, we sound a second note, a Fifth above it, and then a third note, a Major Third above the second. What will be the vibration-fraction of the interval formed

by the first and third notes, *i.e.* of the sum of a Fifth and a Major Third? We will, for shortness, call the three notes (1), (2), (3), in order of ascending pitch. The vibration-fractions being, for (1)—(2), $\frac{3}{2}$, and, for (2)—(3), $\frac{5}{4}$, we proceed thus :—

While (2) makes 4 vibrations, (3) makes 5 vibrations.

Therefore, while (2) makes 1 vibration, (3) makes $\frac{5}{4}$ vibrations.

Therefore, while (2) makes 3 vibrations, (3) makes $3 \times \frac{5}{4}$ vibrations.

But while (2) makes 3 vibrations, (1) makes 2 vibrations.

Therefore, while (1) makes 2 vibrations, (3) makes $3 \times \frac{5}{4}$ vibrations.

Therefore, while (1) makes 1 vibration, (3) makes $\frac{3}{2} \times \frac{5}{4}$ vibrations.

Our result, then, is *the two vibration-numbers multiplied together.*   The reasoning is perfectly general, and gives us the following rule.

To find the vibration-fraction of the sum of two intervals, multiply their separate vibration-fractions together.

96.   Next, take the opposite case.   Let (2) be a Major Third above (1), and (3) a Fifth above (1), and let the vibration-fraction for the interval (2)—(3) be required.

While (1) makes 4 vibrations, (2) makes 5 vibrations.

Therefore, while (1) makes 1 vibration, (2) makes $\frac{5}{4}$ vibrations.

But, while (1) makes 2 vibrations, (3) makes 3 vibrations.

Therefore, while (1) makes 1 vibration, (3) makes $\frac{3}{2}$ vibrations.

Hence, while (2) makes $\frac{5}{4}$ vibrations, (3) makes $\frac{3}{2}$ vibrations.

Therefore, while (2) makes $\frac{1}{4}$ of a vibration, (3) makes $\frac{1}{5} \times \frac{3}{2}$ of a vibration.

Therefore, while (2) makes 1 vibration, (3) makes $\frac{4}{5} \times \frac{3}{2}$ vibrations.

The result here is *the quotient resulting from the division of the larger vibration-fraction by the smaller :* hence we have this general rule :—

To find the vibration-fraction of the difference of two intervals, divide the vibration-fraction of the wider by that of the narrower interval.

Thus *multiplication and division of vibration-fractions* correspond to *addition and subtraction of intervals*[1].

97.   One of the simplest cases of our second rule occurs when an interval has to be inverted. The 'inversion' of any assigned interval narrower than an Octave is the difference between it and an Octave, *i.e.* the interval which remains after it has been subtracted from an Octave.   Thus to find the vibration-number for the inversion of the Minor Third we merely have to divide 2 by $\frac{6}{5}$, or in other words *invert the vibration-fraction of the interval and multiply by* 2.   This applies to all cases.   In the particular example selected, the result is $\frac{5}{3}$; the inversion of the Minor Third is therefore the Major Sixth.   The relation between an interval and its inversion is

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[1] By simply reducing the numerical results, obtained in §§ 95, 96, the student will establish the following propositions :

'A Major Third added to a Fifth gives a Major Seventh.'

'A Major Third subtracted from a Fifth leaves a Minor Third.'

obviously mutual, so that each may be described as the inversion of the other. Accordingly the inversion of the Major Sixth is the Minor Third.

The following table[1] shows the three pairs of consonant intervals narrower than an Octave which stand to each other in the mutual relation of inversions.

Minor Third ($\frac{6}{5}$)—Major Sixth ($\frac{5}{3}$)

Major Third ($\frac{5}{4}$)—Minor Sixth ($\frac{8}{5}$)

Fourth ($\frac{4}{3}$)—Fifth ($\frac{3}{2}$)

98. A combination of musical sounds of different pitch is called a 'chord.' Hitherto we have considered only chords of two notes, or 'binary' chords. We now go on to chords of three notes, or as they are usually called 'triads.' A triad contains *three* intervals, one between its extreme notes, and one between the middle note and *each* of the other two. In order that the chord may be free from dissonance, those intervals must *all three* be concords.

99. We may, then, search for consonant triads in the following manner. Having selected the lowest of the three notes at pleasure, choose two others each of which forms *with the bottom note* a consonant interval. Next, examine whether the interval formed by the last chosen notes *with each other* is also a

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[1] The last two results of this table will be easily verified by the student.

concord. If so, the triad itself is consonant. In order to determine all the consonant triads within an Octave above the fixed bottom note, we must assign to the middle and top notes every possible consonant position with respect to the bottom note, and reject all such positions as give rise to dissonant intervals between those notes themselves. The remaining positions will constitute all the consonant triads which have for their lowest note that originally selected. The intervals at our disposal are, for the middle note from the Minor Third to the Minor Sixth, and, for the upper note, from the Major Third to the Major Sixth.

In the annexed table[1] the possible positions of the middle note with respect to the bottom note are shown in the left-hand vertical column, the name of each interval being accompanied by its vibration-fraction. The possible positions of the top note are similarly shown in the highest horizontal column. Each space common to a horizontal and a vertical column contains the vibration-fraction of the interval formed between the simultaneous positions of the middle and upper notes named at their extremities. When this interval is dissonant its vibration-fraction is enclosed in a square bracket. When it

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[1] Copied with slight modifications from Helmholtz's work.

is a concord the name of the interval is, in each case, appended.

	Major Third $\frac{5}{4}$	Fourth $\frac{4}{3}$	Fifth $\frac{3}{2}$	Minor Sixth $\frac{8}{5}$	Major Sixth $\frac{5}{3}$
Minor Third $\frac{6}{5}$	$\boxed{\frac{25}{24}}$	$\boxed{\frac{10}{9}}$	$\frac{5}{4}$ Major Third	$\frac{4}{3}$ Fourth	$\boxed{\frac{25}{18}}$
Major Third $\frac{5}{4}$		$\boxed{\frac{16}{15}}$	$\frac{6}{5}$ Minor Third	$\boxed{\frac{32}{25}}$	$\frac{4}{3}$ Fourth
Fourth $\frac{4}{3}$			$\boxed{\frac{9}{8}}$	$\frac{6}{5}$ Minor Third	$\frac{5}{4}$ Major Third
Fifth $\frac{3}{2}$				$\boxed{\frac{16}{15}}$	$\boxed{\frac{10}{9}}$
Minor Sixth $\frac{8}{5}$					$\boxed{\frac{25}{24}}$

100.   The following, then, are all the cases;

Middle Note.	Upper Note.
Minor Third	Fifth or Minor Sixth
Major Third	Fifth or Major Sixth
Fourth	Minor Sixth or Major Sixth

or, in Musical notation,
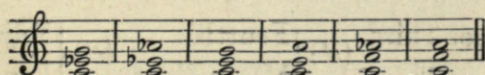
They give us two groups of three triads each, which may be arranged thus :—

(a) $\begin{cases} \text{Fifth.} \\ \text{Major Third.} \end{cases}$ (b) $\begin{cases} \text{Minor Sixth.} \\ \text{Minor Third.} \end{cases}$ (c) $\begin{cases} \text{Major Sixth.} \\ \text{Fourth.} \end{cases}$

(a) $\begin{cases} \text{Fifth.} \\ \text{Minor Third.} \end{cases}$ (β) $\begin{cases} \text{Major Sixth.} \\ \text{Major Third.} \end{cases}$ (γ) $\begin{cases} \text{Minor Sixth.} \\ \text{Fourth.} \end{cases}$

101. Instead of defining our six consonant triads, as is here done, by the intervals formed by their middle and top notes with the bottom note, we may define them by the intervals separating the middle from the bottom note, and the top from the middle note. In order to make this change we have, in each case, a process of subtraction of intervals to perform. Thus the difference between a Fifth and a Major Third is $\frac{3}{2} \times \frac{4}{5}$, *i.e.* $\frac{6}{5}$, or a Minor Third.

Proceeding in this way, we find that the top and middle notes are separated by the following intervals :

(a)	(b)	(c)	(a)	(β)	(γ)
Minor Third	Fourth	Major Third	Major Third	Fourth	Minor Third

Hence we may write our two groups as follows :—

(a') $\begin{cases} \text{Minor Third.} \\ \text{Major Third.} \end{cases}$ (b') $\begin{cases} \text{Fourth.} \\ \text{Minor Third.} \end{cases}$ (c') $\begin{cases} \text{Major Third.} \\ \text{Fourth.} \end{cases}$

$(\alpha')\begin{cases}\text{Major Third.}\\\text{Minor Third.}\end{cases}(\beta')\begin{cases}\text{Fourth.}\\\text{Major Third.}\end{cases}(\gamma')\begin{cases}\text{Minor Third.}\\\text{Fourth.}\end{cases}$

It will now be easy to show that the triads of each group are very closely connected together. Take (α'), and let us form another triad from it by causing its bottom note to ascend one Octave, the other two remaining where they were. The middle will then become the bottom note, the top the middle note, and the Octave of the former bottom note the top note. Hence the lower interval of the new triad will be the upper interval of the old one, *i.e.* a Minor Third. The upper interval of the new triad will necessarily be the *inversion of the interval which separated the extreme notes of the old triad.* This interval is a Fifth [see (a), p. 194], and its inversion, by the table on p. 191, is a Fourth. Hence the new triad is $\begin{cases}\text{Fourth.}\\\text{Minor Third.}\end{cases}$ which is identical with (b').

If we modify (b') in the same way, the new interval is the inversion of the Minor Sixth, *i.e.* the Major Third, and the resulting triad, viz. $\begin{cases}\text{Major Third.}\\\text{Fourth.}\end{cases}$ is identical with (c'). This triad, when similarly treated, brings us back to (a'), and the cycle of changes is complete. By an extension of the word 'inversion,' it is usual to call the triads (b') and (c') the *first and second inversions of the triad* (a').

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Exactly similar relations hold between the members of the second group of triads: (β') and (γ') are accordingly called the first and second inversions of the triad (α'). The proof is exactly like that just given, and will be easily supplied by the reader.

102.   If we choose C as the bottom note of (a') and (α'), these two groups, which are called respectively Major and Minor triads, will be expressed in Musical notation by
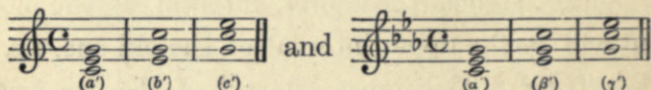
They may also be defined in the language of Thorough-Bass, which refers every chord to its lowest note in accordance with the mode adopted in (a), (b), (c); (a), (β), (γ). Thus the triads (a'), (b'), (c') would be indicated by the figures $\frac{5}{3}$, $\frac{6}{3}$, $\frac{6}{4}$ respectively, and *so would the triads* (α') (β') (γ'); the differences between Minor and Major Thirds and Sixths being left to be indicated by the key-signature.

The positions (a') and (α') are regarded as the fundamental ones of each group; (b'), (c') and (β'), (γ'), being treated as derived from them by successive inversion.

103.   The fundamental triads bear the name of their lowest notes, thus (a') and (α') are called respectively the Major and Minor *common-chords of C.*

The remaining members of each group are not named after their own lowest note, but after that of their fundamental inversion; thus (b'), (c') and (β'), (γ') are respectively Major and Minor common-chords of *C in their first and second inversions.*

Common-chords of more than three constituent sounds can only be formed by adding to the consonant triads notes which are exact Octaves above or below those of the triads.

104.   The marked distinction existing, for every musical ear, between the bright open character of Major, and the gloomy veiled effect of Minor chords, is connected by Helmholtz with the different ways in which combination-tones enter in the two cases. The positions of the first-order combination-tones for each of the six consonant triads are shown in crotchets in the appended stave, the primaries being indicated by minims. Each interval gives rise to its own combination-tone, but, in the cases of the fundamental position and second inversion of the *C*-Major triad, two combination-tones happen to coincide. The reader will at once notice that in the Major group no note extraneous to the harmony is brought in by the combination-tones. In the Minor group this is no longer the case. The fundamental position and the first inversion of the triad both bring in an $A\flat$ which is foreign to the harmony, and the second

inversion involves an additional extraneous note, $B\flat$.
The position of these adventitious sounds is not such
as to produce *dissonance,* for which they are too far
apart from each other and from the notes of the
triad; but they cloud the transparency of the har-
mony and so give rise to effects characteristic of the
Minor mode.
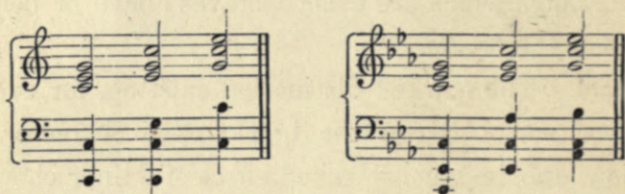
The imperfect nature of Minor compared with
Major triads comes out with peculiar distinctness on
the harmonium; as indeed, from the powerful combi-
nation-tones of that instrument, we should naturally
have anticipated.

ON PURE INTONATION AND TEMPERAMENT.

105. The vibration-fractions of the intervals formed by the notes of the Major scale *with the tonic* are, including the Octave of the tonic, these :—

$$\frac{9}{8}, \quad \frac{5}{4}, \quad \frac{4}{3}, \quad \frac{3}{2}, \quad \frac{5}{3}, \quad \frac{15}{8}, \quad 2.$$

The intervals *between the successive notes of the scale* are determined by dividing each of these fractions by that which precedes it. Thus the consecutive intervals of the Major scale come out as follows :—

C	D	E	F	G	A	B	C'
$\frac{9}{8}$	$\frac{10}{9}$	$\frac{16}{15}$	$\frac{9}{8}$	$\frac{10}{9}$	$\frac{9}{8}$	$\frac{16}{15}$	

Only three different intervals are obtained. $\frac{9}{8}$ is slightly wider than $\frac{10}{9}$; $\frac{16}{15}$ decidedly narrower than the other two. $\frac{9}{8}$ and $\frac{10}{9}$ are called whole Tones, $\frac{16}{15}$ a half-Tone or semi-Tone, though, strictly speaking, two intervals of this width added together somewhat exceed the greater of the two whole Tones; since $\frac{16}{15} \times \frac{16}{15}$ or $\frac{256}{225}$ is to $\frac{9}{8}$ in the ratio of 2048 to 2025.

Suppose we had a keyed instrument containing a number of Octaves, each divided into seven notes forming the ordinary scale as above, so that any Music could be played on it not involving notes foreign to the key of C Major. But now, suppose we wanted to be able to play in *another* Major key as well as in that of C, for instance G. It would be necessary for this purpose to introduce two new notes in every Octave of the key-board. If G is the new tonic, A will not serve as the Second of its scale, because the step between tonic and Second is not $\frac{10}{9}$ but $\frac{9}{8}$. Hence we must have a fresh note lying between A and B. Further, F will not do for the seventh of the scale of G, as it is separated from G by $\frac{9}{8}$ instead of $\frac{16}{15}$. This necessitates a second additional note lying between F and G. If we take as our original Octave that from middle-C upwards, we have the following vibration-numbers :—

C	D	E	F	G	A	B	C'
264	297	330	352	396	440	495	528

The new notes, being respectively $\frac{9}{8}$ *above* and $\frac{16}{15}$ *below* G, have for their vibration-numbers $\frac{9}{8} \times 396$ and $\frac{15}{16} \times 396$, *i.e.* $445\frac{1}{2}$ and $371\frac{1}{4}$. The other notes of the scale of G Major can be supplied from that of C Major. Hence these two scales are closely connected with each other. Another key nearly related to the key of C is that of F. Its Fourth is $\frac{4}{3} \times 352$,

or $469\frac{1}{3}$, which falls between A and B. Its Major Sixth is $\frac{5}{3} \times 352$, or $568\frac{2}{3}$, which is clearly not the exact Octave of any note between C and C'. The corresponding note in that Octave, found by division by 2, is $293\frac{1}{3}$, which comes between C and D. Thus, two more new notes in the Octave must be introduced to make the key of F major attainable.

106. In order that the reader may see at a glance the variety of sounds which are requisite to supply complete Major scales for the seven keys of C, D, E, F, G, A and B, the vibration-numbers for all the notes of these scales are calculated out and exhibited in the following table.

Reducing those notes which lie beyond the

Tonic	Second	Major Third	Fourth	Fifth	Major Sixth	Major Seventh
C, 264	297	330	352	396	440	495
D, 297	$334\frac{1}{8}$	$371\frac{1}{4}$	396	$445\frac{1}{2}$	495	$556\frac{7}{8}$
E, 330	$371\frac{1}{4}$	$412\frac{1}{2}$	440	495	550	$618\frac{3}{4}$
F, 352	396	440	$469\frac{1}{3}$	528	$586\frac{2}{3}$	660
G, 396	$445\frac{1}{2}$	495	528	594	660	$742\frac{1}{2}$
A, 440	495	550	$586\frac{2}{3}$	660	$733\frac{1}{3}$	825
B, 495	$556\frac{7}{8}$	$618\frac{3}{4}$	660	$742\frac{1}{2}$	825	$928\frac{1}{8}$

Octave, by dividing them by 2, and arranging the results in order of magnitude, we have *eleven notes* foreign to the scale of *C* Major, the positions of which, with reference to the notes of that scale, are as follows :—

C, 275, $278\frac{7}{16}$, $293\frac{1}{3}$, D, $309\frac{3}{8}$, E, $334\frac{1}{8}$, F, $366\frac{2}{3}$, $371\frac{1}{4}$, G, $412\frac{1}{2}$, A, $445\frac{1}{2}$, $464\frac{1}{16}$, $469\frac{1}{3}$, B.

107. If it is desired to be able to play in the *Minor* mode of each of the seven keys, as well as in the *Major*, additional notes will be called for. Each scale must contain three Minor intervals, viz. Minor Third, Minor Sixth, and Minor Seventh. The following subsidiary table exhibits the vibration-numbers of the sounds forming these intervals with the successive key-notes.

Tonic	Minor Third	Minor Sixth	Minor Seventh
C, 264	$316\frac{4}{5}$	$422\frac{2}{5}$	$469\frac{1}{3}$
D, 297	$356\frac{2}{5}$	$475\frac{1}{5}$	528
E, 330	396	528	$586\frac{2}{3}$
F, 352	$422\frac{2}{5}$	$563\frac{1}{5}$	$625\frac{7}{9}$
G, 396	$475\frac{1}{5}$	$633\frac{4}{5}$	704
A, 440	528	704	$782\frac{2}{9}$
B, 495	594	792	880

Reducing these to one Octave, as before, we find *seven* notes not included in the previous list, occupying the following positions :—

C, $281\frac{3}{5}$, D, $312\frac{8}{9}$, $316\frac{4}{5}$, E, F, $356\frac{2}{5}$, $391\frac{1}{9}$, G, $422\frac{2}{5}$, A, $475\frac{1}{5}$, B.

108.   Hence, to play perfectly in tune in both Major and Minor modes of the seven keys C, D, E, F, G, A, B, it is necessary to have a key-board with *twenty-five notes* in every Octave.   This number, large as it is, by no means includes all necessary notes.   Modern music is written in *sharp* and *flat* keys, *i.e.* in such whose tonics are not coincident with any one of the notes *CDE...B*.   Moreover, the sharp and flat key-notes are different from each other.   Thus $G\sharp$, being a Major Third above E, is, as the first table shows, $412\frac{1}{2}$; while $A\flat$ is seen, by the second table, to be $422\frac{2}{5}$, which is a somewhat sharper note.   As the seven keys which have been already examined require 25 notes in the Octave, we may anticipate that the *ten* additional sharp and flat keys will bring in a still larger number.

109.   It is, however, needless to institute a detailed inquiry into these scales, as the facts already established amply suffice to show how serious are the imperfections of tune which inevitably beset instruments of fixed sounds, such as the pianoforte, harmonium and organ, containing *only twelve notes in*

each Octave. Pure intonation in the 'natural' keys alone, *i.e.* those whose tonics are white notes on the board, demands, as has been seen, more than twice this number of available sounds; and many more still, if the keys with tonics on the black notes are to be included. Perfect tuning *in all the keys* being entirely out of the question, a compromise of some kind is the only possible course. Thus we may tune a single key, say *C,* perfectly; in which case most of the other keys will be so out of tune as to be unbearable. Or again, we may distribute the errors over certain often-used keys, and accumulate them in others which are of less frequent occurrence.

Expedients of this kind are described as modes of 'tempering,' and the system adopted in tuning any particular instrument is called its 'temperament.' A vast number of different methods of tempering have been proposed and tried during the history of the organ and pianoforte.

110. That which has at last been almost universally adopted is the system of *equal temperament.* It consists in dividing each Octave into *twelve precisely equal intervals.* Each of these intervals is called a semi-Tone, and any two of them together a Tone or whole Tone.

The Octave of which *C,* 264, is the lowest note,

will contain, on the equal temperament system, the sounds set out below. The vibration-numbers are given true to the nearest integer. When a note is slightly sharper than that so indicated, this is shown by the sign + attached to the vibration-number in question ; when slightly flatter, by the sign −. For the sake of comparison, the perfect intervals of the same scale are written below the tempered ones.

C,	C♯,	D,	D♯,	E,	F,	F♯,	G,	G♯,	A,	A♯,	B
264,	280−,	296+,	314−,	333−,	352+,	373+.	395+,	419+,	444−,	470+,	498+

C,		D,	E♭,	E,	F,		G,	A♭,	A,	B♭,	B
264,		297,	317−,	330,	352,		396,	422+,	440,	469+,	495

It is clear that the regions of the tempered scale where the tuning is most imperfect are in the neighbourhood of the Thirds and Sixths. *E* and *A* are nearly three vibrations per second too sharp. The Fourth and Fifth are less out of tune, in fact only wrong by a fraction of a vibration per second.

111. The intervals of the tempered scale are so nearly equal to those of the perfect scale that, when the notes of the former are sounded *successively*, it requires a delicate ear to recognise the defective character of the tuning. When, however, more than one note is heard *at a time*, the case becomes quite different. We saw in Chapter VIII. how rigorously accurate the tuning of a consonant interval must be, to secure the greatest smoothness of which it is capable. Such

intervals were also shown to be generally very closely
bounded by harsh discords. Now since, in the
system of equal temperament, no interval except
that of the Octave is accurately in tune, it follows
that every representative of a concord must be less
smooth than it would be were the tuning perfect.
One of the greatest charms of Music, and especially
of modern Music, lies in the vivid contrast presented
by consonant and dissonant chords in close juxta-
position. Temperament, by impairing, even though
but slightly, the perfection of the concords, necessarily
somewhat weakens this contrast, and takes the edge
off the musical pleasure which, in the hands of a great
composer, it is capable of giving us. A fact already
once adverted to (p. 175) may be again adduced here,
as illustrating the effect of temperament in blurring
distinctions of consonance and dissonance, viz. that
on the key-board of the pianoforte the same two
notes which represent C Ab, which is a concord,
though not a very smooth one, also appear in $C - G\sharp$,
which is a decided discord. A reference to § 108 will
show that, with perfect tuning, $G\sharp$ and Ab are different
notes having vibration-numbers in the proportion of
$412\frac{1}{2}$ to $422\frac{2}{5}$.

One of the readiest ways of recognising the de-
fective character of equal-temperament tuning is,
first to allow a few accurate voices to sing a series

of sustained chords in three or four parts, without accompaniment, and then, after noticing the effect, to let them repeat the phrase while the parts are at the same time played on the pianoforte. The sour character of the concords of the accompanying instrument will be at once decisively manifested. Voices are able to sing perfect intervals, and their clear transparent concords contrast with the duller substitutes provided by the pianoforte in a way obvious to every moderately sensitive ear.

112. Since the voice is endowed with the power of producing all possible shades of pitch within its compass, and thus of singing absolutely pure intervals, it is clear that we ought to make the most of this great gift, and, especially in the case of those persons who are to be public singers, allow during the season of preparation contact with the purest examples of intonation only. Unfortunately the practice of most singing-masters is the very reverse of this. The pupil is systematically accompanied, during vocal practice, on the pianoforte, and thus accustomed to habitual familiarity with intervals which are never strictly in tune. No one can doubt the tendency of such constant association to impair the sensitiveness to minute differences of pitch on which delicacy of musical perception depends. Evil communications are not less corrupting to good

ears than to good manners. I am convinced that we have here the reason why so comparatively few of our trained vocalists, whether amateurs or professionals, are able to sing perfectly in tune. The untutored singing of a child who has never undergone the ear-spoiling process often gives more pleasure by the natural purity of its intonation than the vocalisation of an opera-singer who cannot keep in tune. The remedy is to practise without accompaniment, or with that of an instrument like the violin[1], which is not tied down to a few fixed sounds. Even with the pianoforte something might be done by having it, when intended to be used only in assisting vocal practice, put into perfect tune in one single key, and using that key only.

The services of such an instrument would, no doubt, be comparatively very restricted, but this might not be without a corresponding advantage, if the vocalist were thereby compelled to rely a little

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[1] That a violinist can play *pure* intervals has been established by Professor Helmholtz by the following decisive experiment, performed with the aid of Herr Joachim. A harmonium was employed which had been tuned so as to give pure intervals with certain stops and keys; and tempered intervals with others. A string having been tuned in unison with a common tonic of both systems, it was found that the intervals played by the eminent violinist agreed with those of the *natural,* not with those of the *tempered* scale.

more on his own unaided ear, lay aside his corks and swim out boldly into the ocean of Sound.

113.   The Musical notation in ordinary use evidently takes for granted a scale consisting of a limited number of fixed sounds.   Moreover, it indicates directly *absolute pitch,* and only indirectly *relative pitch.*   In order to ascertain the interval between any two notes on the stave, we must go through a little calculation involving the clef, the key-signature, and perhaps additional 'accidental' sharps or flats.   Now these are complications which, if necessary for pianoforte Music[1], are perfectly gratuitous in the case of vocal Music.   The vocalist wants only to be told on what note to begin, and what *intervals* to sing afterwards, *i.e.* is essentially concerned with absolute pitch *only at the start,* and needs be troubled with it no further.

The established notation thus encumbers the vocalist with information which he does not want, and yet fails to communicate the one special piece of information which he *does* want, viz. the relation which the note to be sung bears to its tonic or key-note. There is nothing in the established notation to mark clearly and directly what this relation is, in each case,

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[1] In a Paper published in Vol. I. of the Proceedings of the Musical Association (London, Chappell and Co.), I have endeavoured to prove that there is no such necessity.

intended to be.   Unless the vocalist, besides his own
'part' is provided with that of the accompaniment,
and possesses some knowledge of Harmony, he can-
not ascertain how the notes set down for him are
related to the key-note and to each other.   The
extreme inconvenience of this must have become
painfully evident to any one who has frequently sung
in concerted Music from a single part.

A Bass, we will suppose, after leaving off on $F\sharp$,
is directed to rest thirteen bars, and then come in
fortissimo on his high $E\flat$.   It is impossible for him
to keep the absolute pitch of F in his head during
this long interval, which is perhaps occupied by
the other voices in modulating into some remote
key ; and his part vouchsafes no indication in what
relation the $E\flat$ stands to the notes, or chords, im-
mediately preceding it.   There remains, then, nothing
for him to do but to sing, at a venture, *some* note
at the top of his voice, in the hope that it may
prove to be E, though with considerable dread, in
the opposite event, of the conspicuous ignominy
entailed by a *fortissimo* blunder.

The essential requisite for a system of vocal
notation therefore is that, whenever it specifies any
sound, it shall indicate, in a direct and simple
manner, the relation in which that sound stands to
its tonic for the time being.   A method by which

this criterion is very completely satisfied shall now be briefly described.

114.   The old Italian singing-masters denoted the seven notes of the Major scale, reckoned from the key-note upwards, by the syllables

<div align="center">do, re, mi, fa, sol, la, si,</div>

pronounced, of course, in the continental fashion. So long as a melody moves only in the Major mode, without 'chromatic' notes or modulation, it clearly admits of being written down, as far as relations of *pitch only* are concerned, by the use of these syllables. The opening phrase of 'Rule Britannia,' for instance, would stand thus:—

sol*, do, do, do, re, mi, fa, sol, do, re, re, mi, fa, mi.

In order to abridge the notation, we may indicate each syllable by its initial letter. The ambiguity which would thus arise between *sol* and *si* is got rid of by altering the latter syllable into *ti*.   In order to distinguish a note from those of the same name in the adjacent Octaves above and below it, an accent is added either above or below the corresponding initial.   Thus *d′* is an Octave above *d* ; *d,* an Octave below *d*.

Where a modulation, *i.e.* a *change of tonic*, occurs, it is shown in the following manner.   A note

* The asterisked sol is to be sung an Octave below that not thus marked.

necessarily stands in a two-fold relation to the out-going and the in-coming tonic. The interval it forms with the new tonic is different from that which it formed with the old one. Each of these intervals can be denoted by a suitable syllable-initial, and the displacement of one of these initials by the other represents in the aptest manner the super-session of the old by the new tonic. The old initial is written above and to the left of the new one. Thus *^df* indicates that the note *re* is to be *sung*, but *its name changed* to *fa*. As this is a somewhat difficult point a few instances are appended, ex-pressed both in the established notation and in that now under consideration. The modulations selected are, from *C* to *G* ; from *C* to *F* ; from *E* to *C* ; from *G* to *F♯*.
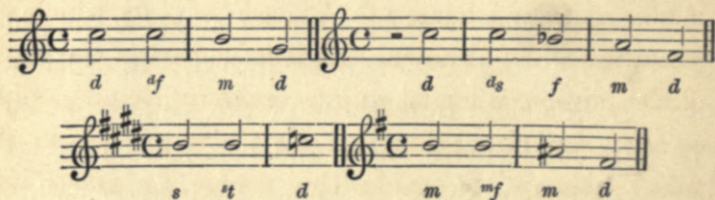
Immediately after a modulation the ordinary syl-lable-initials come into use again, and are employed until a fresh modulation occurs. It will be seen at once, that the difficulty of 'remote keys,' which is so serious in the established notation, here altogether disappears. For instance, a vocal phrase from Spohr's

'Last Judgment' which in the established notation
stands as follows :—
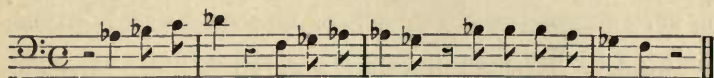
takes, in the notation before us, the perfectly simple
form,

$$s\ l\ t\ |\ d'\ m\ f\ s\ |\ s\ f\ l\ l\ l\ s\ |\ f\ m.$$

As another example, take the following from the
same work :—

f   r   r   s   t,   d   d   r   r   f   t,   d

115.   The system of notation of which a cursory
sketch has just been given originated with the late
Miss S. A. Glover of Norwich, but has received its
present form at the hands of Mr J. Curwen, to whom
it also owes the name of 'Tonic Sol-fa,' by which it
is now so widely known.   As it is no part of the plan
of the present work to go into technical details, only
so much has been said about Mr Curwen's system as
was necessary to enable the reader to grasp its
essential principle.   No mention has been made of
the notation for Minor and Chromatic intervals, nor
of that for denoting the relations of time by measures
appealing directly to the eye, instead of by mere
symbols.   On these and all other points connected
with his system, Mr Curwen's published works on

Tonic Sol-Fa give full and throughly lucid and intelligible explanations. Mr Curwen has also created a very extensive literature of the best vocal Music, printed in his own notation; given a most remarkable impulse to choral singing; and established a system of graded certificate-examinations guaranteeing the attainment of corresponding stages of Musical cultivation[1].

I have enjoyed some opportunities of watching the progress of beginners taught on the old system, and on that of the Tonic Sol-Fa, and I assert without the slightest hesitation that, as an instrument for imparting the power of sight-singing, the new system is enormously, overwhelmingly, superior to the old. In fact, I am prepared to maintain that the complicated repulsiveness of the pitch-notation in the old system must be held mainly responsible for the humiliating fact that, of the large number of musically well-endowed persons of the opulent classes who have undergone an elaborate vocal training, comparatively few are able to sing even the very simplest Music *at sight*. Set a young lady thus instructed to sing a psalm-tune she has never seen before, and we all know what the result is likely to be. Now, there is no more inherent difficulty in teaching a child

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[1] Mr Curwen died on May 26, 1880. His son, Mr J. Spencer Curwen, is now at the head of the Tonic Sol-Fa movement.

with a fairly good ear to *sing* at sight, than there is in making him *read* ordinary print at sight. A vocalist who can sing only a few elaborately prepared songs ought to be regarded as on a level with a school-boy who should be unable to read except out of his own book. If evidence be wanted to make good this assertion, it is at once to hand in the fact that the youngest children, when well trained on the Tonic Sol-Fa system, soon obtain a power of steady and accurate sight-singing, and will even tell you whether a new tune pleases them after merely looking it over without uttering a sound.

The reader is requested to observe that the above remarks are strictly limited to the achievements of the Tonic Sol-Fa system in *vocal* Music. I express no opinion as to the applicability of its notation to *instrumental* Music, nor do I wish to maintain that even in the vocal branch it has arrived at perfection. On the contrary, I am doubtful whether its time-notation, when applied to very complicated rhythmic divisions, does not become more difficult than the system in ordinary use, and I consider the notation adopted for the Minor mode to be unquestionably defective. On the main point, however, viz. the decisive superiority of its pitch-notation over that of the established system, and the vitally important consequences as to purity of intonation which neces-

sarily follow from this superiority, I desire to express the most confident and uncompromising opinion.[1]

116.   In closing the inquiry which occupies the preceding chapters, it will be advisable to examine very concisely the bearing of our principal result, the theory of consonance and dissonance, on the æsthetics of music.   Dissonance was shown to arise from rapid beats, and the concords were classed in order, according to their more or less complete freedom from dissonance; the Octave coming first, followed by the Fifth, Fourth, Major Third and Sixth and Minor Third and Sixth.   This classification was strictly *physical,* depending exclusively on smoothness of effect.   On its own ground, therefore, this classification is absolutely unassailable, and whoever says, for instance, that a Major Third is a smoother concord than a Fifth or Octave, asserts what is as demonstrably false as that the moon goes round the earth in an exact circle.   Nevertheless, it by no means necessarily follows that the smoothest concords must be the most gratifying to the ear.   There may be some *other* property of an interval which gives us greater satisfaction than mere consonance.

[1] These topics will be found handled somewhat more at large in a Lecture delivered by me at Manchester on Jan. 11, 1883. ("The Tonic Sol-fa Movement" London, J. Curwen and Sons, 8 Warwick Lane, E. C.)

Assuming, for the moment, that such a property does in fact exist, the ear, if called on to arrange the consonant intervals in the order of their *pleasantness*, might very well bring out a different arrangement from that adopted by physical science on grounds of smoothness alone. Æsthetic considerations come in here with the same right to be heard that mechanical considerations possess within their own domain.

117. Now unquestionably the ear's order of merit is *not* the same as the mechanical order. It places Thirds and Sixths first, then the Fourth and Fifth, and the Octave last of all. The constant recurrence of Thirds and Sixths in two-part Music, compared with the infrequent employment of the remaining concords, leaves no doubt on this point. In fact these intervals have a peculiar charm about them, not possessed by the Fourth or Fifth to anything like the same extent, and by the Octave not at all.

The thin effect of the Octave undoubtedly depends on the fact that every partial-tone of the higher of two clangs forming that interval coincides exactly with a partial-tone of the lower clang. Thus *no new sound* is introduced by the higher clang; the quality of that previously heard is merely modified by the alteration of relative intensity among its constituent partial-tones. Major and

Minor Thirds bring in a greater variety of pitch in
the resulting mass of sound than does the Fifth; but
this can hardly be said of the Major and Minor
Sixths compared with the Fourth.   On the whole, I
am inclined to attribute the predilection of the ear
for Thirds and Sixths over the other concords to
circumstances connected with its perception of *key-
relations,* though I am not able to give a satisfactory
account of them.   The ear enjoys, in alternation
with consonant chords, dissonances of so harsh a
description as to be barely endurable when sustained
by themselves.   As passages containing such extreme
discords take the following.   The chord marked *
should in each case be played *first by itself,* and then

Last Chorus, Bach's
"Passion" (St. Matthew).

in the place assigned to it by the composer.   The
effect of the isolated discord is so intensely harsh,

that it is at first difficult to understand how any preceding and succeeding concords can make it at all tolerable; yet the sequence, in both the phrases cited, is of the rarest beauty.

Considerations such as those just alleged tend to show that, while physical science is absolutely authoritative in all that relates to the constitution of musical sounds and the smoothness of their combinations, the composer's direct perception of what is musically beautiful must mainly direct him in the employment of his materials. It would be a serious error to force upon him a number of rules planned, on scientific principles, to secure the maximum smoothness of effect ; since mere smoothness is often a matter of extremely secondary importance compared with grandeur of harmony and masterly movement of parts. The nature of the subject may sometimes call for such treatment as shall secure exceptional smoothness. In such a case the rules may become of considerable importance. It is well, therefore, that a composer should know and be able to handle them, but he should never allow them to fetter his freedom in wielding the higher and more spiritual weapons of his warfare.